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Electric/Magnetic Duality with Gauge Singlets

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Abstract

We demonstrate how gauge singlets can be used to find new examples of Kutasov duality (i.e. where the matching of the dual theories relies on a non-zero superpotential) in $\mathcal{N} = 1$ $SU(N)$ SQCD with F_Q flavours of quark and multiple generations of adjoints, or antisymmetrics, or symmetric. The role of the singlets is to simplify greatly the truncation of the chiral ring whilst maintaining an R -symmetry, and at the same time allowing an unambiguous identification of the elementary mesons of the magnetic theory. The dual theories satisfy all the usual tests, including the highly non-trivial 't Hooft anomaly matching conditions.

1 Introduction and background

An interesting possibility in the context of electric-magnetic duality [1] is that the supersymmetric Standard Model is the magnetic dual of some unknown electric theory [2, 3, 4]. This would have a bearing on many important questions. For example it would accommodate Landau poles in models of direct gauge mediation of supersymmetry (SUSY) breaking. The dynamical scales could also explain hierarchies in the SUSY breaking sector and even in the Yukawa couplings. More recently [4] it was argued that this possibility may, rather counterintuitively, be compatible with unification. That argument was based on known examples of Kutasov duality, i.e. duality that relies on a non-zero superpotential [5]. The vacuum structure of these models was analysed in the simplest SQCD plus a single adjoint case by Kutasov, Schwimmer and Seiberg (KSS); it can be that of a broken GUT theory with electric and magnetic dual descriptions [6, 7]. Further related studies of these theories were made in Refs.[8, 9, 10, 11].

The observation made in Ref.[4] was that (as shown in Figure 1.1) unification in such theories is preserved in the mapping from electric to magnetic descriptions, with the gauge couplings of the magnetic theory appearing to unify at the same energy scale but at unphysical (imaginary) values of the gauge couplings. This unphysical gauge unification also happens in a supersymmetric Standard Model with a large number of messengers in complete $SU(5)$ representations, and so in Ref.[4] it was suggested that the latter would be a strong hint of a magnetic dual GUT theory. Furthermore it was argued that this would explain why the supersymmetric Standard Model appears to unify, but the proton does not decay. The proton decay operator (being generated at the GUT scale) is a baryon operator that is generated in the electric theory. However the decay takes place at low energies and is therefore computed in the magnetic description with the relevant operator being mapped to the corresponding magnetic baryon: consequently the decay rate is suppressed by the many powers of Λ/M_{GUT} associated with the mapping of electric to magnetic baryons.

Unfortunately the arguments of Ref.[4] are limited by the fact that no convincing electric dual description of the supersymmetric Standard Model or indeed the Georgi-Glashow $SU(5)$ model is known. In particular, in for example an $SU(5)$ framework, one would like to be able to build chiral models, models with three generations, with adjoint GUT Higgs fields and antisymmetrics and so on. Although some of these ingredients (for example chiral models) are present in the models in the literature (related examples with $SU(N)$ gauge groups can be found in Refs.[7, 12, 13, 14]), the particle content is very constrained and is far from that of the Georgi-Glashow model. Motivated by this fact, we will in this paper demonstrate how one can extend the Kutasov class of electric-magnetic dual theories using gauge and flavour singlets.

In very general terms our approach can be described as follows. First recall the three basic tests of electric-magnetic duality [15]: the two dual theories should

- have the same moduli space of vacua, and in particular a one-to-one map between the degrees of freedom represented by meson and baryon operators,
- they should share the same global symmetries and have the same global anomalies (i.e. the 't Hooft anomaly matching conditions),
- their moduli spaces should still match under all deformations of the theories by chiral operators.

The KSS-like theories can be shown to pass all these tests, thanks to the IR dynamics being simplified by the addition to the superpotential of various operators. In its original formulation, the KSS theory consisted of SQCD plus a single adjoint, X , and the superpotential operator

$$W \supset X^{k+1} \tag{1.1}$$

for some integer k . This term alters the infra-red dynamics (it is a dangerously irrelevant operator, as one says) for appropriate choices of $SU(N)$ and F_Q flavours, and truncates the chiral ring, allowing one to match the degrees of freedom in the moduli space relatively easily. Further

examples of this kind of duality were discovered in Refs.[13, 14] involving more adjoint fields, and/or symmetric and antisymmetric representations. An obstacle restricts the whole class of models however: as more fields are added, the chiral ring becomes more difficult to truncate and additional operators have to be added to the superpotential to do this; on the other hand each new operator projects out some global symmetry and weakens the 't Hooft anomaly matching test. In particular once the R -symmetry is explicitly broken the 't Hooft anomaly matching test becomes virtually meaningless, so this symmetry at least one needs to preserve. Our approach in order to overcome this problem is to use an additional singlet to enable us to add the extra operators needed to truncate the chiral ring while preserving a meaningful R -symmetry. The role of this singlet is to break the R -symmetry spontaneously by acquiring a VEV, thereby generating the R -symmetry breaking couplings necessary to truncate the chiral ring.

At first sight one might suppose that there is little to be gained from doing this. Indeed the R -charged couplings (and hence the singlet) will generally appear in the elementary mesons that are a feature of the spectrum of the magnetic dual theory, so it might seem that the R -charges in the magnetic theory are going to be arbitrary, and the correct anomaly matching condition impossible to identify. This is not so however: the matching of the composite mesons of the electric description to the elementary mesons of the magnetic one entails an assignment of singlet contribution that is unambiguous. Moreover a nontrivial test is that the equivalent assignment in the baryons makes their global charges match automatically. With the R -charges of the elementary mesons of the magnetic dual theory duly fixed, anomaly matching can then proceed as normal, and it provides the usual powerful test of the duality.

In the following section we begin by showing how this can be done in the original KSS (SQCD plus single-adjoint) model of Refs.[6, 7]. An important feature of this model is as we have stated the possibility of adding deformations that would break the GUT symmetry. Although these operators are not crucial for truncating the chiral ring (being relevant deformations rather than drastically changing the IR behaviour as the original X^{k+1} operator did), it is still interesting to see how they can be made compatible with a preserved R -symmetry by the addition of a gauge singlet. The procedure required to identify the elementary mesons of the magnetic description in this simple case will serve as a useful guide for later models. We find that anomaly matching in the deformed theory works in two cases; the first corresponds to the original deformed KSS model with the singlet VEV merely parameterizing the size of the deformation. The second model is different from the original model, and as we shall see is (unlike the first) not associated with the flow to a fixed point.

In sections 3, 5 and 6 we consider models with two or more adjoints and models with extra antisymmetric and symmetric representations respectively. In each case we can add to the superpotential enough operators to truncate the chiral ring while still maintaining an R -symmetry and therefore highly non-trivial 't Hooft anomaly matching conditions.

Section 4 deals in more detail with the RG flow. Concentrating on the adjoint models, we construct a complete picture of the flow in models with one adjoint plus a singlet using a -maximization [16, 17]. The additional singlet alters the RG flow, and gives various fixed points; the flow is to the KSS model with the singlet appearing in a deformation term, however the theory can flow between fixed points corresponding to different deformations. We use this picture to deduce the RG flow behaviour for the more general models with additional adjoints. For the models with an odd number of adjoint fields, we argue that the endpoint of the flow is also the deformed KSS model. Models with an even number of adjoints flow to normal SQCD. In both cases the duality at the endpoints of the flow is well understood.

We close the Introduction by making a parenthetical remark about the phenomenology. Recall that our ultimate goal is to be able to describe the supersymmetric Standard Model (or a Georgi-Glashow like GUT theory) as a magnetic dual. These theories generally do not allow anomaly-free global R -symmetries because of the complexity of their Yukawa couplings and Higgs mass terms. It is therefore likely that any R -symmetry would have to be spontaneously broken anyway in order to have reasonable phenomenology, and using gauge singlets to do this would be the simplest way. We consider this to be an additional motivation for the study in this paper.

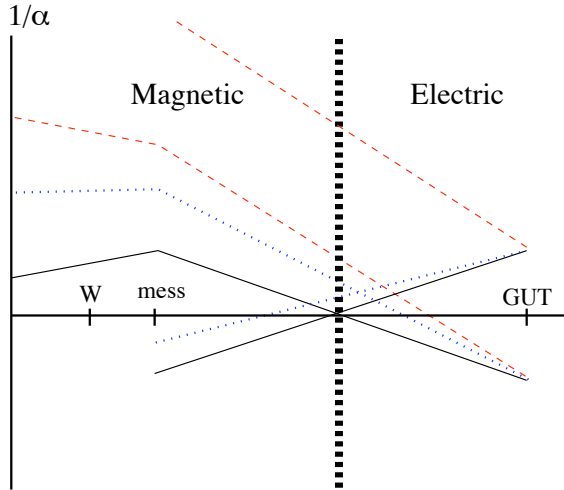


Figure 1.1: *The dual-unification scenario of Ref.[4]: the supersymmetric Standard Model appears to run to unphysical gauge unification when there are many messengers in complete $SU(5)$ multiplets. This is mapped to a real unification occurring in an electric dual description that is valid above the Landau pole scale.*

2 R -symmetry in the deformed KSS model

Let us first revisit the model of KSS in order to see how singlets can be introduced. As we have said, one of the appealing features of those models was the ability to match the electric and magnetic theories under deformations of the electric superpotentials

$$W_{\text{el}} = \sum_{i=0}^{k-1} \frac{t_i}{k+1-i} \text{Tr} [X^{k+1-i}] + \lambda \text{Tr} [X] \quad (2.1)$$

where X is an adjoint field of the $SU(N)$ gauge group and we have chosen our basis for X such that there is no X^k term as explained in Ref.[7]. The undeformed theory has only the t_0 term. The $t_{i>0}$ deformations spontaneously break the gauge symmetry and lead to a rich vacuum structure; it is then possible to show that there exists a similar deformation in the magnetic theory that produces the same vacuum. There is an aspect of this procedure that we will address in this section as a warm-up exercise which is this. By adding the extra terms in Eq.(2.1), one breaks the R -symmetries of the model. In principle therefore the 't Hooft anomaly matching conditions apply only to the undeformed theory. However it is often useful to think of couplings such as $t_{i>0}$ as background R -charged fields that acquire VEVs. Thus a way to match the anomalies directly in the deformed theory would be to consider these fields as singlets in the spectrum and to do the anomaly matching on the complete theory. The singlet VEVs can then be fixed at the end to generate the required R -breaking terms in Eq.(2.1) spontaneously. This is what we will investigate here.

We commence by summarizing the models of KSS [6, 7] in more detail. They are based on an $SU(N)$ gauge group with F_Q flavours of quarks and anti-quarks, and an adjoint field of the $SU(N)$ denoted by X . The symmetry content is

$$SU(F_Q)_L \times SU(F_Q)_R \times U(1)_B \times U(1)_{R_1} \times U(1)_{R_2}. \quad (2.2)$$

When there is only the t_0 term in the superpotential, the global symmetry is partially broken but retains a $U(1)_R$ symmetry. The matter content is then summarised by Table 2.1. When there are also non-zero t_i couplings, the $U(1)_R$ -symmetry is completely broken. The F -term equation for

	$SU(N)$	$SU(F_Q)_L$	$SU(F_Q)_R$	$U(1)_B$	$U(1)_R$
Q	\mathbf{N}	$\mathbf{F_Q}$	$\mathbf{1}$	$\frac{1}{N}$	$1 - \frac{2}{k+1} \frac{N}{F_Q}$
\tilde{Q}	$\overline{\mathbf{N}}$	$\mathbf{1}$	$\overline{\mathbf{F_Q}}$	$-\frac{1}{N}$	$1 - \frac{2}{k+1} \frac{N}{F_Q}$
X	\mathbf{adj}	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{2}{k+1}$

Table 2.1: *The matter content of the electric theory in the KSS model.*

the adjoint in this case can easily be solved by diagonalizing the X using $SU(N)$ rotations; the equation for a single entry χ on the diagonal is

$$W' = 0 \equiv \sum_{i=0}^{k-1} t_i \chi^{k-i} + \lambda. \quad (2.3)$$

This is a k 'th order polynomial so there are k roots: hence

$$\langle X \rangle = \begin{pmatrix} \chi_1 \mathbf{I}_{p_1} & & & \\ & \chi_2 \mathbf{I}_{p_2} & & \\ & & \ddots & \\ & & & \chi_k \mathbf{I}_{p_k} \end{pmatrix} \quad \text{where} \quad \sum_{i=1}^k p_i = N. \quad (2.4)$$

The gauge symmetry is broken as

$$SU(N) \rightarrow SU(p_1) \times SU(p_2) \dots SU(p_k) \times U(1)^{k-1}. \quad (2.5)$$

Note that the parameter λ is used as a Lagrange multiplier to fix $\text{Tr}[X] = 0$.

To get the corresponding magnetic theory we need to identify a set of elementary meson fields associated with composite operators of the electric model. A crucial aspect of the superpotential is that it truncates the chiral ring; that is the equation of motion for X (in for example the undeformed theory) sets $X^k = 0$ (ignoring the Lagrange multiplier term) along the F -flat directions. This means that when matching the moduli spaces, one need only consider operators up to X^{k-1} (in the deformed theory X^k is mapped to a polynomial in X of order less than k). Thus there are k types of meson operator that we denote m_j ;

$$m_j = \tilde{Q} X^j Q, \quad j = 0, \dots, k-1. \quad (2.6)$$

The $j = 0$ object is the meson of usual Seiberg duality. Indeed, the $k = 1$ model is just the original Seiberg SQCD model if one integrates out the adjoint field. The field content of the magnetic theory is q, \tilde{q}, m_j and x , where x is an adjoint in the magnetic gauge group, and where the elementary magnetic mesons are directly and unambiguously identified with the composite operator m_j . Assuming for the moment that there is only the t_0 term in the electric theory then, as in Refs.[6, 7], the baryon matching implies that the gauge group of the full (unbroken) magnetic theory is

$$SU(n) = SU(kF_Q - N). \quad (2.7)$$

The matter content is summarised in Table 2.2. The superpotential in the deformed magnetic theory is of the form

$$W_{\text{mag}} = - \sum_{i=0}^{k-1} \frac{t_i}{k+1-i} \text{Tr}[x^{k+1-i}] + \frac{1}{\mu^2} \sum_{l=0}^{k-1} t_l \sum_{j=0}^{k-1-l} m_j \tilde{q} x^{k-1-j-l} q. \quad (2.8)$$

This form of superpotential was deduced in Ref.[7] by equating the vacuum structures of the electric and magnetic theories. The broken model in the magnetic theory is

$$SU(n) \rightarrow SU(F_Q - p_1) \times SU(F_Q - p_2) \dots SU(F_Q - p_k) \times U(1)^{k-1}. \quad (2.9)$$

	$SU(n)$	$SU(F_Q)_L$	$SU(F_Q)_R$	$U(1)_B$	$U(1)_R$
q	\mathbf{n}	$\overline{\mathbf{F_Q}}$	$\mathbf{1}$	$\frac{1}{n}$	$1 - \frac{2}{k+1} \frac{n}{F_Q}$
\tilde{q}	$\overline{\mathbf{n}}$	$\mathbf{1}$	$\mathbf{F_Q}$	$-\frac{1}{n}$	$1 - \frac{2}{k+1} \frac{n}{F_Q}$
x	\mathbf{adj}	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{2}{k+1}$
m_j	$\mathbf{1}$	$\mathbf{F_Q}$	$\overline{\mathbf{F_Q}}$	0	$2 - \frac{4}{k+1} \frac{N}{F_Q} + \frac{2j}{k+1}$

Table 2.2: *The matter content of the magnetic theory in the KSS model; $n = kF_Q - N$.*

Note that the matching of the sub-theories $SU(p_i) \leftrightarrow SU(F_Q - p_i)$ is generically the standard SQCD duality.

Let us now turn to the anomaly matching of the deformed theories and, as we have proposed, introduce a singlet field ϕ whose role is to generate the t_i couplings by acquiring a VEV: we therefore write $t_i \rightarrow \phi^{\rho_i}$ for some exponents ρ_i . The coefficients in the magnetic superpotential are directly related to those in the electric one which, in turn, directly relates the singlet structure in the magnetic theory to that in the electric one. As a simple example consider the electric superpotential

$$W_{\text{el}} = \phi^{\rho_0} \text{Tr} \left[\frac{X^{k+1}}{k+1} \right] + \phi^{\rho_l} \text{Tr} \left[\frac{X^{k+1-l}}{k+1-l} \right]. \quad (2.10)$$

We will henceforth omit the Lagrange multiplier term, and also assume $k > 2$ and $l > 1$. We may read off the magnetic superpotential from Eq.(2.8):

$$\begin{aligned} W_{\text{mag}} = & -\phi^{\rho_0} \text{Tr} \left[\frac{x^{k+1}}{k+1} \right] - \phi^{\rho_l} \text{Tr} \left[\frac{x^{k+1-l}}{k+1-l} \right] + \\ & \left(\frac{\phi^{\rho_0}}{\mu^2} \sum_{j=0}^{k-1} m_j \tilde{q} x^{k-1-j} q + \frac{\phi^{\rho_l}}{\mu^2} \sum_{j=0}^{k-1-l} m_j \tilde{q} x^{k-1-j-l} q \right). \end{aligned} \quad (2.11)$$

Note that all the couplings of the magnetic theory, in particular the powers of ϕ , are already determined by the vacuum structure. However the parameter μ^2 is an unknown free parameter that itself carries R -charge. For example, it determines the matching conditions for the dynamical scales of the electric and magnetic theories (Λ and $\bar{\Lambda}$ respectively) which are [6]

$$\Lambda^{2N-F_Q} \bar{\Lambda}^{2n-F_Q} = \left(\frac{\mu}{t_0} \right)^{2F_Q}. \quad (2.12)$$

The R -symmetry is anomaly free (with respect to the mixed gauge- R anomalies) so the dynamical scales do not carry R -charge or depend on ϕ . To be consistent the R -charge of μ has to be $\rho_0 R_\phi$ and moreover μ must scale as $\mu \sim \phi^{\rho_0}$. Both electric and magnetic theories now have an R -symmetry that is spontaneously broken by the VEV of ϕ .

Indeed let us now lay out the charges of the fields explicitly. By virtue of the electric superpotential and the absence of mixed $U(1)_R \times SU(N)^2$ anomalies, the matter content of the electric theory is given by Table 2.3. Since we know the charges of the mesons from Eq.(2.6) we can independently derive the magnetic superpotential by writing down terms consistent with the R -symmetry. This turns out to yield precisely W_{mag} as required, so that the superpotential deduced from the vacuum structure is consistent with that deduced from the R -symmetries. At the classical level W_{mag} allows arbitrary values of ρ_0 and ρ_l . However we will now see that the 't Hooft anomaly matching conditions constrain them.

To do the anomaly matching we first need to identify correctly the elementary fields of the magnetic theory, and in particular deal with some arbitrariness about the definition of the elementary mesons. Specifically, the mesons' role is to mirror (and project out via their equations of

	$SU(N)$	$SU(F_Q)_L$	$SU(F_Q)_R$	$U(1)_B$	$U(1)_R$
Q	\mathbf{N}	$\mathbf{F_Q}$	$\mathbf{1}$	B_Q	$1 - \frac{2N(\rho_l - \rho_0)}{F_Q[\rho_l(1+k) - \rho_0(1+k-l)]}$
\tilde{Q}	$\overline{\mathbf{N}}$	$\mathbf{1}$	$\overline{\mathbf{F_Q}}$	$-B_Q$	$1 - \frac{2N(\rho_l - \rho_0)}{F_Q[\rho_l(1+k) - \rho_0(1+k-l)]}$
X	\mathbf{adj}	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{2(\rho_l - \rho_0)}{\rho_l(1+k) - \rho_0(1+k-l)}$
ϕ	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{2l}{\rho_l(1+k) - \rho_0(1+k-l)}$

Table 2.3: The matter content of the electric theory in the deformed KSS model. B_Q is arbitrary (conventionally it is taken to be the baryon number, i.e. $B_Q = 1/N$).

motion) the corresponding directions in the moduli space of the magnetic theory. For example, consider the leading meson term of the magnetic superpotential. Remembering that $\mu \sim \phi^{\rho_0}$ this can be written as

$$\phi^{-\rho_0} \sum_{j=0}^{k-1} m_j \tilde{q} x^{k-1-j} q \equiv \phi^{-\rho_0} \sum_{j=0}^{k-1} \left(\tilde{Q} X^j Q \right) (\tilde{q} x^{k-1-j} q) \quad (2.13)$$

where we have exhibited the composite objects of the electric theory that are the erstwhile elementary mesons. The equations of motion of m_j therefore project out the corresponding directions in the magnetic moduli space. The arbitrariness lies in the singlet content of what we call the elementary meson of the magnetic theory. Because the singlets carry R -charge, unless we determine this correctly, anomaly matching is impossible.

We therefore propose the following principle: the singlets are to be distributed among the elementary and composite magnetic mesons in the same way. That is, rewriting the leading interaction as

$$\phi^{-\rho_0} \sum_{j=0}^{k-1} m_j \tilde{q} x^{k-1-j} q \equiv \sum_{j=0}^{k-1} \left(\tilde{Q} (\phi^\alpha X)^j Q \right) (\tilde{q} (\phi^\alpha x)^{k-1-j} q), \quad (2.14)$$

where

$$\alpha = -\frac{\rho_0}{k-1}, \quad (2.15)$$

allows us to identify the elementary mesons M_j of the magnetic theory as

$$M_j \equiv \tilde{Q} (\phi^\alpha X)^j Q. \quad (2.16)$$

As we have said, identifying this singlet contribution to the meson for non-zero ρ_0 is an important step since it fixes the R -charges of the elementary mesons and hence the anomaly matching.

An independent check of this identification of the magnetic mesons is that the same identification is appropriate for the mapping between electric and magnetic baryons. In detail, the baryons of the electric theory in the deformed KSS model are given by

$$B_{\{r_j\}} \sim \prod_{j=0}^{k-1} (X^j Q)^{r_j} = Q^N X^{\sum_{j=0}^{k-1} j r_j} \quad \text{where} \quad \sum_{j=0}^{k-1} r_j = N \quad (2.17)$$

(for concision we have omitted colour indices, which are contracted with a rank N epsilon tensor). The magnetic baryons to which these are mapped were found (by mass flow and by equating the global charges including R -charge) in Refs.[6, 7] to be

$$b_{\{r_j\}} \sim \frac{t_0^{kF_Q/2}}{(\mu^2)^{-n/2}} \prod_{j=0}^{k-1} (x^j q)^{F_Q - r_j} \sim (\phi^{\rho_0})^{(kF_Q - 2n)/2} q^n x^{\sum_{j=0}^{k-1} j(F_Q - r_j)}. \quad (2.18)$$

Using $\sum_{j=0}^{k-1} j (F_Q - r_j) = \sum_{j=0}^{k-1} j r_j - (k-1) F_Q$, we find that the R -charges of the baryons are equal if

$$NR_Q - nR_q + (k-1) \left(N - \frac{kF_Q}{2} \right) R_X + \rho_0 \left(n - \frac{kF_Q}{2} \right) R_\phi = 0 \quad (2.19)$$

which is satisfied for any ρ_0 (the specific R -symmetry of the undeformed models [6, 7] is, of course, equivalent to taking $\rho_0 = 0$). Using Eq.(2.15) one finds that this is equivalent to

$$NR_Q - nR_q + (\alpha R_\phi + R_X) (k-1) \left(N - \frac{kF_Q}{2} \right) = 0. \quad (2.20)$$

In other words, the prefactor

$$\frac{t_0^{kF_Q/2}}{(\mu^2)^{-n/2}} \sim (\phi^{\rho_0})^{(kF_Q-2n)/2} \quad (2.21)$$

which is required to match the R -charges of the electric and magnetic baryons, is equivalent to associating with each X precisely the same factor ϕ^α as we did for the mesons

$$\begin{aligned} B_{\{r_j\}} &\sim \prod_{j=0}^{k-1} [(\phi^\alpha X)^j Q]^{r_j} = Q^N (\phi^\alpha X)^{\sum_{j=0}^{k-1} j r_j} \\ b_{\{r_j\}} &\sim \prod_{j=0}^{k-1} [(\phi^\alpha x)^j q]^{F_Q - r_j} = q^n (\phi^\alpha x)^{\sum_{j=0}^{k-1} j (F_Q - r_j)}. \end{aligned} \quad (2.22)$$

All that remains is to determine the $U(1)_B$ charges of the magnetic quarks. They can be fixed by demanding that the $U(1)_B$ charges of the electric and magnetic baryons match up:

$$B_q = -B_{\bar{q}} = \frac{N}{n} B_Q. \quad (2.23)$$

We may now proceed to anomaly matching. The mixed anomalies are found to be

$$\begin{aligned} U(1)_B \times SU(F_Q)_L^2 &: NB_Q \\ U(1)_B \times SU(F_Q)_R^2 &: -NB_Q \\ U(1)_R \times SU(F_Q)_L^2 &: -\frac{2N^2(\rho_l - \rho_0)}{F_Q[\rho_l(1+k) - \rho_0(1+k-l)]} \\ U(1)_R \times SU(F_Q)_R^2 &: -\frac{2N^2(\rho_l - \rho_0)}{F_Q[\rho_l(1+k) - \rho_0(1+k-l)]} \\ U(1)_B \times U(1)_R^2 &: 0 \\ U(1)_R \times U(1)_B^2 &: -\frac{4N^2 B_Q^2(\rho_l - \rho_0)}{\rho_l(1+k) - \rho_0(1+k-l)} \\ U(1)_B &: 0 \\ U(1)_R &: \frac{(2N^2 + k + 3)(\rho_0 - \rho_l) - l(\rho_0 - 2)}{[\rho_l(1+k) - \rho_0(1+k-l)]}, \end{aligned} \quad (2.24)$$

in both theories. The last two correspond to mixed $U(1)$ -gravity anomalies – there is a contribution from the diagrams with gravitinos in the loop that is obviously universal and has been omitted. Three of the cubic anomalies also match

$$\begin{aligned} SU(F_Q)_L^3 &: N \\ SU(F_Q)_R^3 &: -N \\ U(1)_B^3 &: 0. \end{aligned} \quad (2.25)$$

Up to this point the matching occurs for arbitrary ρ_0 and ρ_l . However, the final $U(1)_R^3$ anomaly only matches for the three combinations of ρ_0 and ρ_l that are presented in Table 2.4. There is

ρ_0	ρ_l	$U(1)_R^3$
0	any	$N^2 - 1 - \frac{16N^4}{F_Q^2(1+k)^3} + (N^2 - 1) \left(\frac{2}{1+k} - 1 \right)^3 + \left(\frac{2l}{\rho_l(1+k)} - 1 \right)^3$
any	$\left(\frac{k-l-1}{k-1} \right) \rho_0$	$-F_Q^{-2} \rho_0^{-3} [2N^4 \rho_0^3 + F_Q^2 ([k-1]^3 + 3\rho_0[k-1]^2 + 3\rho_0^2[k-1] - \rho_0^3[N^2 - 2])]$
any	ρ_0	$-\rho_0^{-3} (\rho_0 - 2)^3$

Table 2.4: Choices of the exponents ρ_0 and ρ_l that realize anomaly matching in the deformed KSS model with singlet.

also the special case $n = N$ which satisfies the anomaly matching for any ρ_0 and ρ_l . Note that, because of its complexity, the existence of simple solutions to the matching of the $U(1)_R^3$ anomalies is certainly not a foregone conclusion.

In the first case the R -symmetry is clearly that of the original KSS model with the singlet acting to preserve it under the addition of a deformation, but it is interesting that there are two other consistent choices. The second case

$$\rho_l = \left(\frac{k-l-1}{k-1} \right) \rho_0 \quad (2.26)$$

is equivalent to

$$R_X = 1 \quad \text{and} \quad R_\phi = \frac{1-k}{\rho_0}. \quad (2.27)$$

The last case has $\rho_l = \rho_0$, ergo $R_X = 0$. It is therefore less interesting, since the singlet has simply introduced an R -symmetry of its own under which none of the matter superfields are charged, and will be discarded.

The simplest choice of exponents in the $\rho_0 \neq 0$ model (where $\rho_l = \rho_0 - l\rho_0/(k-1)$) is the superpotential

$$W_{\rho_0 \neq 0} = \phi^{k-1} \text{Tr} \left[\frac{X^{k+1}}{k+1} \right] + \phi^{k-1-l} \text{Tr} \left[\frac{X^{k+1-l}}{k+1-l} \right] \quad (2.28)$$

with $R_X = 1$ and $R_\phi = -1$. Note that as far as the anomalies are concerned, we are at liberty to choose $l = k+1$ which generates a constant in the superpotential

$$W_{\rho_0 \neq 0} = \phi^{k-1} \text{Tr} \left[\frac{X^{k+1}}{k+1} \right] + \phi^{-2} \quad (2.29)$$

when ϕ gets a VEV. Such a term could conceivably be generated dynamically. In the $\rho_0 = 0$ model the R -symmetry is precisely that of the KSS model. In this case ρ_l is unconstrained and the superpotential is

$$W_{\rho_0=0} = \text{Tr} \left[\frac{X^{k+1}}{k+1} \right] + \phi^{\rho_l} \text{Tr} \left[\frac{X^{k+1-l}}{k+1-l} \right] \quad (2.30)$$

with $R_\phi = \frac{2l}{\rho_l(k+1)}$. Clearly both cases allow for generally deformed superpotentials with more than two terms; i.e. the general R -symmetric superpotentials (with these particular R -charges) can be written

$$\begin{aligned} W_{\rho_0 \neq 0} &= \phi^{k-1} \text{Tr} \left[\frac{X^{k+1}}{k+1} f(\phi X) \right] \\ W_{\rho_0=0} &= \text{Tr} \left[\frac{X^{k+1}}{k+1} f(\phi^{\rho_l/l} X^{-1}) \right] \end{aligned} \quad (2.31)$$

where f is an arbitrary polynomial of order $< k+1$.

There are various limits that one can consider. In the $\rho_0 = 0$ theory for example, one can continuously take the $\phi \rightarrow 0$ limit. This simply corresponds to turning off the deformation of the KSS model. The magnetic superpotential is

$$W_{\text{mag}} = \text{Tr} \left[\frac{x^{k+1}}{k+1} \right] + \phi^{\rho_l} \text{Tr} \left[\frac{x^{k+1-l}}{k+1-l} \right] + \frac{1}{\mu^2} \sum_{j=0}^{k-1} M_j (\tilde{q} x^{k-1-j} q) + \frac{\phi^{\rho_l}}{\mu^2} \sum_{j=0}^{k-1-l} M_j (\tilde{q} x^{k-1-j-l} q) \quad (2.32)$$

and goes continuously to the undeformed model as required. In the opposite $\phi \rightarrow \infty$ limit, we may define a rescaled $\hat{X} = \phi^{\rho_l/(k+1-l)} X$ and similarly rescaled magnetic adjoints, so that the electric superpotential becomes

$$W_{\rho_0=0} = \phi^{-\rho_l(k+1)/(k+1-l)} \text{Tr} \left[\frac{\hat{X}^{k+1}}{k+1} \right] + \text{Tr} \left[\frac{\hat{X}^{k+1-l}}{k+1-l} \right] \quad (2.33)$$

and the magnetic superpotential is

$$W_{\text{mag}} = \phi^{-\rho_l(k+1)/(k+1-l)} \text{Tr} \left[\frac{\hat{x}^{k+1}}{k+1} \right] + \text{Tr} \left[\frac{\hat{x}^{k+1-l}}{k+1-l} \right] + \frac{\phi^{-\rho_l(k+1)/(k+1-l)}}{\hat{\mu}^2} \sum_{j=0}^{k-1} \hat{M}_j (\tilde{q} \hat{x}^{k-1-j} q) + \frac{1}{\hat{\mu}^2} \sum_{j=0}^{k-1-l} \hat{M}_j (\tilde{q} \hat{x}^{k-1-j-l} q) \quad (2.34)$$

where

$$\hat{M}_j = \tilde{Q} \hat{X}^j Q \quad \text{and} \quad \hat{\mu} = \phi^{\rho_l/(k+1-l)} \mu. \quad (2.35)$$

Thus in the large ϕ limit, keeping $\hat{\mu}$ constant, we have KSS duality with $k \rightarrow k-l$. In terms of the original scales this corresponds to dialing down the Landau poles with respect to μ . Taking, for example, $\bar{\Lambda} \sim \Lambda$, the matching relation tells us that

$$\Lambda = \frac{\mu}{t_0} \sim \phi^{-\rho_l/(k+1-l)} \hat{\mu} \quad (2.36)$$

in this case. We shall later consider the RG flow in the theory.

3 Models with more adjoints

Now that we have demonstrated the use of singlets in electric/magnetic duality, we can go on to apply this technique to models for which the dualities are not currently known. All of the models presented below have electric theories based on SQCD, with gauge group $\text{SU}(N)$ and chiral quark flavour symmetry $\text{SU}(F_Q)$. We will try always to denote fields and parameters in upper case for the electric theory and in lower case for the magnetic theory. In each case the magnetic gauge group is $\text{SU}(n)$ for some value of n which must be determined.

3.1 Two adjoints

The first model we consider extends SQCD with two $\text{SU}(N)$ adjoints X and Y , and an $\text{SU}(N)$ singlet ϕ . Electric/magnetic duality with two adjoints was first discussed by Brodie and Strassler in Refs.[12, 13], however the form of their model is much more restricted than that which we are about to present. As our electric superpotential we take

$$W_{\text{el}} = \phi^{\rho_X} \text{Tr} \left[\frac{X^{k_X+1}}{k_X+1} \right] + \phi^{\rho_Y} \text{Tr} \left[\frac{Y^{k_Y+1}}{k_Y+1} \right] + \text{Tr} [XY]. \quad (3.1)$$

for some ρ_X and ρ_Y , which allows a non-anomalous, global symmetry group

$$\text{SU}(F_Q)_L \times \text{SU}(F_Q)_R \times \text{U}(1)_B \times \text{U}(1)_R \quad (3.2)$$

where $U(1)_R$ is a unique R -symmetry. To be exhaustive, we will consider both positive and negative values of ρ_X and ρ_Y (with negative values possibly being generated dynamically).

The F -terms for the adjoints give

$$\begin{aligned}\phi^{\rho_X} X^{k_X} + Y &= 0 \\ \phi^{\rho_Y} Y^{k_Y} + X &= 0.\end{aligned}\tag{3.3}$$

These equations truncate the chiral ring for all non-zero values of ϕ . To see this, one uses the first equation to set $Y = -\phi^{\rho_X} X^{k_X}$ then substitutes into the second equation to find

$$(-1)^{k_Y} \phi^{\rho_X k_Y + \rho_Y} X^{k_X k_Y} = -X.\tag{3.4}$$

Hence truncation occurs at $X^{k_X k_Y}$ or, equivalently, $Y^{k_X k_Y}$.

Eq.(3.4) resembles the usual truncation equation in KSS duality (see §2). It is natural to suppose therefore that the duality proceeds in the same way, but with the effective value of k

$$k^* = k_X k_Y.\tag{3.5}$$

This implies the meson structure

$$m_j \sim \tilde{Q} X^j Q, \quad j = 0, \dots, k^* - 1\tag{3.6}$$

and consequently a dual gauge group $SU(n)$ with

$$n = k^* F_Q - N.\tag{3.7}$$

Note that here it is convenient to use one of the adjoints X to be ‘active’; thus $m_{ak_X+b} \sim \tilde{Q} X^{ak_X+b} Q \sim \tilde{Q} Y^a X^b Q$, where $a < k_Y$ and $b < k_X$.

Let us now investigate the theory a little more closely. The non-anomalous global symmetries and the matter content of the electric theory are displayed in Table 3.1. If one assumes the dual

	$SU(N)$	$SU(F_Q)_L$	$SU(F_Q)_R$	$U(1)_B$	$U(1)_R$
Q	N	F_Q	1	$\frac{1}{N}$	$1 - \frac{N}{F_Q}$
\tilde{Q}	\bar{N}	1	\bar{F}_Q	$-\frac{1}{N}$	$1 - \frac{N}{F_Q}$
X	adj	1	1	0	$\frac{2(\rho_X k_Y + \rho_Y)}{\rho_X(1+k_Y) + \rho_Y(1+k_X)}$
Y	adj	1	1	0	$\frac{2(\rho_Y k_X + \rho_X)}{\rho_X(1+k_Y) + \rho_Y(1+k_X)}$
ϕ	1	1	1	0	$\frac{2(1-k^*)}{\rho_X(1+k_Y) + \rho_Y(1+k_X)}$

Table 3.1: *The matter content of the electric theory in the two adjoint model.*

gauge group Eq.(3.7), the equivalent information for the magnetic theory is as given in Table 3.2, with the full properties of the elementary magnetic mesons yet to be determined. The magnetic $U(1)_B$ charges are again fixed by the baryon map which we will discuss shortly. Following KSS duality, Table 3.2 also assumes that the magnetic superpotential takes the form

$$W_{\text{mag}} = \phi^{\rho_X} \text{Tr} \left[\frac{x^{k_X+1}}{k_X+1} \right] + \phi^{\rho_Y} \text{Tr} \left[\frac{y^{k_Y+1}}{k_Y+1} \right] + \text{Tr}[xy] + \text{meson terms}.\tag{3.8}$$

The gauge singlet ϕ does not interact with the gauge group so is oblivious to the duality and must retain the same charges throughout.

Now let us turn to the elementary mesons of the magnetic theory. In §2 we found that powers of ϕ appeared in their definition, and the same occurs here. To calculate the appropriate powers

	$SU(n)$	$SU(F_Q)_L$	$SU(F_Q)_R$	$U(1)_B$	$U(1)_R$
q	\mathbf{n}	$\overline{\mathbf{F_Q}}$	$\mathbf{1}$	$\frac{1}{n}$	$1 - \frac{n}{F_Q}$
\tilde{q}	$\overline{\mathbf{n}}$	$\mathbf{1}$	$\mathbf{F_Q}$	$-\frac{1}{n}$	$1 - \frac{n}{F_Q}$
x	\mathbf{adj}	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{2(\rho_X k_Y + \rho_Y)}{\rho_X(1+k_Y) + \rho_Y(1+k_X)}$
y	\mathbf{adj}	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{2(\rho_Y k_X + \rho_X)}{\rho_X(1+k_Y) + \rho_Y(1+k_X)}$
ϕ	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{2(1-k^*)}{\rho_X(1+k_Y) + \rho_Y(1+k_X)}$

Table 3.2: *The matter content of the magnetic theory in the two adjoint model, sans elementary mesons. $n = k^* F_Q - N$.*

of ϕ we once again write $M_j = \tilde{Q}(\phi^\alpha X)^j Q$ for $j = 0, \dots, k^* - 1$ and some α to be determined, and then consider the meson terms in the magnetic superpotential. These are

$$W_{\text{meson}} = \sum_{j=0}^{k^*-1} M_j \tilde{q} (\phi^\alpha x)^{k^*-1-j} q \quad (3.9)$$

where the power of x is again shorthand for the corresponding power of $x^{1\dots k_X}$ and $y^{1\dots k_Y}$. Recall that no explicit μ^2 parameter is required now since all the R -charge is carried by the singlets. Note that the magnetic mesons still mirror the electric mesons, hence both use the same value of α . Demanding that all of the terms have R -charge $+2$ leads to a unique solution for α

$$\alpha = \frac{\rho_X + \rho_Y k_X}{1 - k^*}. \quad (3.10)$$

The final form for the electric mesons is therefore

$$M_j = \tilde{Q} \left(\phi^{(\rho_X + \rho_Y k_X)/(1-k^*)} X \right)^j Q. \quad (3.11)$$

Equivalently, we could have used Y instead of X as the active adjoint to describe the mesons, in which case we would have found

$$M_j = \tilde{Q} \left(\phi^{(\rho_Y + \rho_X k_Y)/(1-k^*)} Y \right)^j Q. \quad (3.12)$$

As usual, these mesons are to be added into the magnetic theory as elementary fields. Their properties are summarised in Table 3.3.

	$SU(n)$	$SU(F_Q)_L$	$SU(F_Q)_R$	$U(1)_B$	$U(1)_R$
M_j	$\mathbf{1}$	$\mathbf{F_Q}$	$\overline{\mathbf{F_Q}}$	0	$2 \left(1 + j - \frac{N}{F_Q} \right)$

Table 3.3: *The elementary mesons of the magnetic theory in the two adjoint model.*

We can now test the duality with 't Hooft anomaly matching. As before the $U(1)_B^3$, $U(1)_B \times U(1)_R^2$ and $U(1)_B$ anomalies are all zero; henceforth we will take them as read. The non-zero

mixed anomalies are

$$\begin{aligned}
\mathrm{U}(1)_B \times \mathrm{SU}(F_Q)_L^2 &: 1 \\
\mathrm{U}(1)_B \times \mathrm{SU}(F_Q)_R^2 &: -1 \\
\mathrm{U}(1)_R \times \mathrm{SU}(F_Q)_L^2 &: -N^2/F_Q \\
\mathrm{U}(1)_R \times \mathrm{SU}(F_Q)_R^2 &: -N^2/F_Q \\
\mathrm{U}(1)_R \times \mathrm{U}(1)_B^2 &: -2 \\
\mathrm{U}(1)_R &: -2 - N^2 + \frac{2(1-k^*)}{\rho_X(1+k_Y) + \rho_Y(1+k_X)}.
\end{aligned} \tag{3.13}$$

They are the same in both the electric and magnetic theories. The cubic anomalies all match: the non-zero ones are

$$\begin{aligned}
\mathrm{SU}(F_Q)_L^3 &: N \\
\mathrm{SU}(F_Q)_R^3 &: -N \\
\mathrm{U}(1)_R^3 &: -2 + N^2 - 2N^4/F_Q^2 + 3R_\phi - 3R_\phi^2 + R_\phi^3
\end{aligned} \tag{3.14}$$

with R_ϕ as given in Table 3.1. As the anomalies match for all values of ρ_X and ρ_Y , any values giving well defined R -charges are allowed, and there is no constraint coming from the $\mathrm{U}(1)_R^3$ anomaly. This is because the XY terms mean that $R_X = 2 - R_Y$ and consequently the X and Y fermions have equal and opposite R -charge and don't contribute here. The quark R -charges are independent of ρ_X and ρ_Y for the same reason (i.e. their R -charges are determined by the required absence of $\mathrm{SU}(n)^2 \times \mathrm{U}(1)_R$ anomalies and this gets no contribution from X and Y either). The only field whose R -charge depends on ρ_X and ρ_Y is therefore the singlet which contributes the same in both electric and magnetic theories. Note that demanding a well defined R -charge for the singlet eliminates the solution $\rho_X = \rho_Y = 0$.

The point $\phi = 0$ is singular. Assuming ρ_X and ρ_Y are both positive and that therefore α is negative, then the F -term equations Eq.(3.3) have solution $X = Y = 0$ at $\phi = 0$, and the chiral ring truncates trivially. The behaviour of the $\phi = 0$ theory and no adjoint mass is not well understood, but it is obvious that here ultimately the electric theory flows to that corresponding to normal SQCD with no adjoints, since we can integrate out the adjoint fields X and Y below their (Dirac) mass. The assignment of mesons other than M_0 also breaks down at this point. Correspondingly the $\phi = 0$ theory is ill-defined in the magnetic superpotential, which contains

$$W_{\text{meson}} = \sum_{j=0}^{k^*-1} M_j \tilde{q} (x\phi^\alpha)^{k^*-1-j} q, \tag{3.15}$$

and which therefore diverges. This is hardly surprising because we are using ϕ to encode both the couplings in the superpotential *and* the dimensionful parameter that defines the elementary mesons in the magnetic description (i.e. the equivalent of the μ^2 parameter of the KSS model).

3.1.1 Baryon matching

The form of the mesons Eq.(3.11) strongly suggests that the adjoints should also come with these powers of the singlets in the baryon map between electric and magnetic theories. Applying this observation we construct electric baryons

$$B_{\{r_j\}} = \prod_{j=0}^{k^*-1} \left[(\phi^\alpha X)^j Q \right]^{r_j} \tag{3.16}$$

and magnetic baryons

$$b_{\{r_j\}} = \prod_{j=0}^{k^*-1} \left[(\phi^\alpha x)^j q \right]^{F_Q - r_j} \tag{3.17}$$

with $\sum_{j=0}^{k_X^*-1} r_j = N$ and α as in Eq.(3.10). Again we are for convenience using X to be the ‘active’ adjoint; thus the dressed quarks correspond as $X^{ak_X+b}Q \sim Y^a X^b Q$, where $a < k_Y$ and $b < k_X$. Our experience with KSS duality then leads us to propose the baryon map $B_{\{r_j\}} \leftrightarrow b_{\{r_j\}}$. This map is totally consistent with all global symmetries; in particular the powers of the singlets are precisely those needed to match the R -charges of the baryons.

3.1.2 Generalising the mass term

It may not always be desirable for the electric superpotential Eq.(3.1) to contain the mass term XY . We will therefore outline how this term can be generalised. In order to do this we shall consider the matching of the two theories in the absence of this term, and then deduce that many alternative terms would be consistent with the duality. These terms would not necessarily truncate the chiral ring in which case a simple classical matching of the moduli spaces would not be possible. Nevertheless the fact that all the other tests of the duality are still satisfied is compelling.

In the absence of the XY term, Eq.(3.3) are modified and the chiral ring does not truncate in the same way. Furthermore, the R -symmetry is relaxed to

$$R_Q = R_{\tilde{Q}} = 1 - \frac{N}{F_Q} (R_X + R_Y - 1), \quad R_X = \frac{2 - \rho_X R_\phi}{k_X + 1}, \quad R_Y = \frac{2 - \rho_Y R_\phi}{k_Y + 1} \quad (3.18)$$

for arbitrary R_ϕ in the electric theory and

$$R_q = R_{\tilde{q}} = 1 - \frac{n}{F_Q} (R_x + R_y - 1), \quad R_x = \frac{2 - \rho_X R_\phi}{k_X + 1}, \quad R_y = \frac{2 - \rho_Y R_\phi}{k_Y + 1} \quad (3.19)$$

in the magnetic theory. These facts lead us to reconsider the meson structure of the theory.

The new modification of Eq.(3.3) means that we are generally unable to define an ‘active’ adjoint as we did previously, so are forced to consider the more general meson

$$M_{ab} = \tilde{Q}(\phi^{\alpha_X} X)^a (\phi^{\alpha_Y} Y)^b Q \quad (3.20)$$

with $a = 0, \dots, k_X - 1$ and $b = 0, \dots, k_Y - 1$. The parameters α_X and α_Y are undetermined functions of R_ϕ . The meson terms in the magnetic superpotential now look like

$$W_{\text{meson}} = \sum_{a=0}^{k_X-1} \sum_{b=0}^{k_Y-1} M_{ab} \tilde{q} (\phi^{\alpha_X} x)^{k_X-1-a} (\phi^{\alpha_Y} y)^{k_Y-1-b} q. \quad (3.21)$$

Demanding that these terms all have R -charge 2 allows one to write $\alpha_Y = \alpha_Y(\alpha_X, R_\phi)$ as a function of α_X and R_ϕ , but we are *not* able to remove the R_ϕ dependence of the α ’s. We thus conclude that the meson terms in the magnetic superpotential are inconsistent with more than one R -symmetry. Hence if we remove the XY term from the electric superpotential we are forced to replace it with a different third term that yields a single, non-anomalous R -symmetry in order to maintain the duality. The precise value of R_ϕ is completely arbitrary as long as it is non-zero. We thus have a very large amount of freedom in choosing the third term in the electric superpotential.

Having fixed the R -symmetry and the meson structure there is still one free parameter left in the theory; α_X . The only anomalies that might be sensitive to α_X are those involving the R -symmetry. It turns out that the linear $U(1)_R$ and the mixed $U(1)_R \times SU(F_Q)^2$ anomaly matching does not depend on α_X . The cubic R -symmetry anomaly matching, however, depends quadratically on α_X . If we choose one of the two values of α_X that set the cubic R -symmetry to zero all anomalies therefore match.

Using generalised dressed quarks we can construct electric baryons

$$B_{\{r_{ab}\}} = \prod_{a=0}^{k_X-1} \prod_{b=0}^{k_Y-1} \left[(\phi^{\alpha_X} X)^a (\phi^{\alpha_Y} Y)^b Q \right]^{r_{ab}} \quad (3.22)$$

and magnetic baryons

$$b_{\{r_{ab}\}} = \prod_{a=0}^{k_X-1} \prod_{b=0}^{k_Y-1} \left[(\phi^{\alpha_X} x)^a (\phi^{\alpha_Y} Y)^b q \right]^{F_Q - r_{ab}} \quad (3.23)$$

with $\sum_{a=0}^{k_X-1} \sum_{b=0}^{k_Y-1} r_{ab} = N$. The baryon map $B_{\{r_{ab}\}} \leftrightarrow b_{\{r_{ab}\}}$ is then consistent with all global symmetries for any choice of α_X . The only aspect of the theory that now needs testing is the meson sector. We have not established how (or even if) the chiral ring truncates when the XY term is replaced by some other interaction, which makes it difficult to see whether the mesonic degrees of freedom match up on either side of the proposed duality. However, the fact that all other tests of duality, including the highly non-trivial 't Hooft anomaly matching and the baryon matching (itself, even less trivial than it was in the presence of the XY term) are still passed suggests that the mesons will match up. It may be that we have to resort to investigating the quantum chiral ring, rather than the classical chiral ring we have considered thus far, to see this.

3.2 Three adjoints

The electric theory of this model is similar to the electric theory of the two adjoint model in §3.1, but with an extra $SU(N)$ adjoint Z and an extended superpotential

$$W_{\text{el}} = \phi^{\rho_X} \text{Tr} \left[\frac{X^{k_X+1}}{k_X+1} \right] + \phi^{\rho_Y} \text{Tr} \left[\frac{Y^{k_Y+1}}{k_Y+1} \right] + \phi^{\rho_Z} \text{Tr} \left[\frac{Z^{k_Z+1}}{k_Z+1} \right] + \text{Tr} [XY] + \text{Tr} [YZ]. \quad (3.24)$$

For arbitrary values of ρ_Y , the electric theory has no non-anomalous R -symmetry. One must choose

$$\rho_Y = \frac{\rho_X (1 - k_Y k_Z) - \rho_Z (1 - k_X k_Y)}{k_Z - k_X} \quad (3.25)$$

in order for the theory to possess one. This is markedly different from the two adjoint case in which all ρ 's were independent. With ρ_Y fixed as above, the non-anomalous, global symmetry group is once again

$$SU(F_Q)_L \times SU(F_Q)_R \times U(1)_B \times U(1)_R. \quad (3.26)$$

The F -terms for the adjoints give

$$\begin{aligned} \phi^{\rho_X} X^{k_X} + Y &= 0 \\ \phi^{\rho_Y} Y^{k_Y} + X + Z &= 0 \\ \phi^{\rho_Z} Z^{k_Z} + Y &= 0. \end{aligned} \quad (3.27)$$

One can use the first equation to set $Y = -\phi^{\rho_X} X^{k_X}$, then the second equation to set $Z = -X - (-1)^{k_Y} \phi^{\rho_X k_Y + \rho_Y} X^{k_X k_Y}$. Upon substitution, the third equation reduces down to

$$\phi^{\rho_Z} \left[-X - (-1)^{k_Y} \phi^{\rho_X k_Y + \rho_Y} X^{k_X k_Y} \right]^{k_Z} = \phi^{\rho_X} X^{k_X} \quad (3.28)$$

i.e.

$$X^{k_X k_Y k_Z} \sim \mathcal{O} \left(X^{k_X k_Y k_Z - k_X k_Y + 1} \right). \quad (3.29)$$

As long as $\phi \neq 0$ this implies that the chiral ring is truncated at $X^{k_X k_Y k_Z}$ or, equivalently, $Z^{k_X k_Y k_Z}$. It is not generally possible to use Y as the 'active' adjoint as one is unable to express both X and Z purely in terms of Y .

Using X as the active adjoint (the following argument would be equivalent if we used Z instead) we now follow the procedure of §3.1. Defining

$$k^* = k_X k_Y k_Z \quad (3.30)$$

we ought to be able to use the dual gauge group $SU(n)$ with

$$n = k^* F_Q - N. \quad (3.31)$$

The mesons are

$$M_j = \tilde{Q} (\phi^\alpha X)^j Q, \quad j = 0, \dots, k^* - 1 \quad (3.32)$$

but with a new value of α

$$\alpha = \frac{\rho_Z (k^* - k_X) - \rho_X (k^* - k_Z)}{(1 - k^*) (k_Z - k_X)} \quad (3.33)$$

The magnetic superpotential is

$$\begin{aligned} W_{\text{mag}} = & \phi^{\rho_X} \text{Tr} \left[\frac{x^{k_X+1}}{k_X+1} \right] + \phi^{\rho_Y} \text{Tr} \left[\frac{y^{k_Y+1}}{k_Y+1} \right] + \phi^{\rho_Z} \text{Tr} \left[\frac{z^{k_Z+1}}{k_Z+1} \right] \\ & + \text{Tr} [xy] + \text{Tr} [yz] + \sum_{j=0}^{k^*-1} M_j \tilde{q} (\phi^\alpha x)^{k^*-1-j} q \end{aligned} \quad (3.34)$$

as expected. We summarise the matter content for the electric and magnetic theories in Tables 3.4 and 3.5 respectively. All U(1) charge assignments can be seen to be consistent with both the meson structure and the magnetic superpotential as required.

	SU(N)	SU(F _Q) _L	SU(F _Q) _R	U(1) _B	U(1) _R
Q	N	F_Q	1	$\frac{1}{N}$	$1 - \frac{2N(\rho_Z - \rho_X)}{F_Q[\rho_Z(1+k_X) - \rho_X(1+k_Z)]}$
\tilde{Q}	$\overline{\mathbf{N}}$	1	$\overline{\mathbf{F}_Q}$	$-\frac{1}{N}$	$1 - \frac{2N(\rho_Z - \rho_X)}{F_Q[\rho_Z(1+k_X) - \rho_X(1+k_Z)]}$
X	adj	1	1	0	$\frac{2(\rho_Z - \rho_X)}{\rho_Z(1+k_X) - \rho_X(1+k_Z)}$
Y	adj	1	1	0	$\frac{2(\rho_Z k_X - \rho_X k_Y)}{\rho_Z(1+k_X) - \rho_X(1+k_Z)}$
Z	adj	1	1	0	$\frac{2(\rho_Z - \rho_X)}{\rho_Z(1+k_X) - \rho_X(1+k_Z)}$
ϕ	1	1	1	0	$-\frac{2(k_Z - k_Y)}{\rho_Z(1+k_X) - \rho_X(1+k_Z)}$

Table 3.4: *The matter content of the electric theory in the three adjoint model.*

	SU(n)	SU(F _Q) _L	SU(F _Q) _R	U(1) _B	U(1) _R
q	n	$\overline{\mathbf{F}_Q}$	1	$\frac{1}{n}$	$1 - \frac{2n(\rho_Z - \rho_X)}{F_Q[\rho_Z(1+k_X) - \rho_X(1+k_Z)]}$
\tilde{q}	$\overline{\mathbf{n}}$	1	F_Q	$-\frac{1}{n}$	$1 - \frac{2n(\rho_Z - \rho_X)}{F_Q[\rho_Z(1+k_X) - \rho_X(1+k_Z)]}$
x	adj	1	1	0	$\frac{2(\rho_Z - \rho_X)}{\rho_Z(1+k_X) - \rho_X(1+k_Z)}$
y	adj	1	1	0	$\frac{2(\rho_Z k_X - \rho_X k_Y)}{\rho_Z(1+k_X) - \rho_X(1+k_Z)}$
z	adj	1	1	0	$\frac{2(\rho_Z - \rho_X)}{\rho_Z(1+k_X) - \rho_X(1+k_Z)}$
ϕ	1	1	1	0	$-\frac{2(k_Z - k_Y)}{\rho_Z(1+k_X) - \rho_X(1+k_Z)}$
M _j	1	F_Q	$\overline{\mathbf{F}_Q}$	0	$2R_Q + j(R_X + \alpha R_\phi)$

Table 3.5: *The matter content of the magnetic theory in the three adjoint model, $n = k^* F_Q - N$.*

The 't Hooft anomaly matching reveals another significant difference between the two adjoint

and three adjoint models. The non-zero mixed anomalies are

$$\begin{aligned}
\mathrm{U}(1)_B \times \mathrm{SU}(F_Q)_L^2 & : 1 \\
\mathrm{U}(1)_B \times \mathrm{SU}(F_Q)_R^2 & : -1 \\
\mathrm{U}(1)_R \times \mathrm{SU}(F_Q)_L^2 & : -2N^2 (\rho_Z - \rho_X) / F_Q [\rho_Z (1 + k_X) - \rho_X (1 + k_Z)] \\
\mathrm{U}(1)_R \times \mathrm{SU}(F_Q)_R^2 & : -2N^2 (\rho_Z - \rho_X) / F_Q [\rho_Z (1 + k_X) - \rho_X (1 + k_Z)] \\
\mathrm{U}(1)_R \times \mathrm{U}(1)_B^2 & : -4 (\rho_Z - \rho_X) / [\rho_Z (1 + k_X) - \rho_X (1 + k_Z)] \\
\mathrm{U}(1)_R & : \frac{(\rho_X - 2) k_Z - (\rho_Z - 2) k_X + (\rho_X - \rho_Z) (3 + 2N^2)}{[\rho_Z (1 + k_X) - \rho_X (1 + k_Z)]}.
\end{aligned} \tag{3.35}$$

They are still the same in both the electric and magnetic theories. The cubic flavour anomalies are

$$\begin{aligned}
\mathrm{SU}(F_Q)_L^3 & : N \\
\mathrm{SU}(F_Q)_R^3 & : -N
\end{aligned} \tag{3.36}$$

in both theories; the $\mathrm{U}(1)_B^3$ anomaly is zero. However, the cubic R -symmetry anomalies do not immediately match as per the single adjoint model. Their full forms are formidable, but one can write the difference as

$$f_{3X}(\rho, k) [\rho_Z - \rho_X] [\rho_Z (1 - k_X) - \rho_X (1 - k_Z)] [\rho_Z (k_X - k^*) - \rho_X (k_Z - k^*)] \tag{3.37}$$

where

$$f_{3X}(\rho, k) = \frac{4k^* F_Q (1 - 2k^*) (k^* F_Q - 2N)}{(1 - k^*) [\rho_Z (1 + k_X) - \rho_X (1 + k_Z)]^3} \tag{3.38}$$

When Eq.(3.37) is equal to zero all cubic anomalies match. Solving for ρ_Z yields three possibilities;

$$\rho_Z = \rho_X, \quad \rho_Z = \frac{k_Z - 1}{k_X - 1} \rho_X \quad \text{or} \quad \rho_Z = \frac{k^* - k_Z}{k^* - k_X} \rho_X \tag{3.39}$$

which, using Eq.(3.25), result in

$$\rho_Y = -\rho_X k_Y, \quad \rho_Y = \frac{k_Y - 1}{k_X - 1} \rho_X \quad \text{or} \quad \rho_Y = -\frac{k_Y k^* - 1}{k^* - k_X} \rho_X \tag{3.40}$$

respectively. Any choice of the ρ 's satisfying both Eq.(3.39), Eq.(3.40) and giving well defined R -charges is then allowed. Note that demanding a well defined R -charge for the singlet once again eliminates the solution $\rho_X = \rho_Y = \rho_Z = 0$.

Of particular interest is the second of these solutions. Choosing $\rho_X = k_X - 1$ results in

$$\rho_X = k_X - 1, \quad \rho_Y = k_Y - 1, \quad \rho_Z = k_Z - 1 \tag{3.41}$$

where we have used Eq.(3.25) to calculate ρ_Y . Furthermore, one finds

$$\begin{aligned}
R_Q = R_{\tilde{Q}} & = 1 - N/F_Q \\
R_X = R_Y = R_Z & = 1 \\
R_\phi & = -1
\end{aligned} \tag{3.42}$$

and the simple solution $\alpha = -1$. This solution will be useful when we move on to consider theories with more adjoints.

As final check, we can consider the baryon matching. This is simple and proceeds exactly as in §3.1.1. It remains independent of the values of the ρ 's and the k 's.

The point $\phi = 0$ is again worth noting. The three solutions of Eq.(3.39) allow for four possibilities. ρ_X and ρ_Z must always have the same sign but Eq.(3.40) tells is that ρ_Y can have either. If $\rho_X, \rho_Y, \rho_Z > 0$ or $\rho_X, \rho_Y, \rho_Z < 0$ the F -term equations Eq.(3.27) do not truncate the chiral ring for $\phi = 0$. The same is true for $\rho_X, \rho_Z > 0, \rho_Y < 0$. However, if $\rho_X, \rho_Z < 0, \rho_Y > 0$ the second of Eq.(3.27) gives $Z = -X$ at $\phi = 0$. The first and third of Eq.(3.27) then only have finite field value solutions at $\phi = 0$ if $X^{k_X} = X^{k_Z} = 0$ and $Y = 0$. The chiral ring is thus truncated at the lowest of k_X and k_Z . The electric theory is effectively reduced down to the deformed KSS model.

3.3 F_X adjoints

After §3.1 and §3.2 we expect the same principle will be extendable to an electric theory containing F_X adjoints X_i with $i = 1, \dots, F_X$. Rather than go through the whole process with undetermined ρ_i 's, as we did in the previous sections, we will explicitly demonstrate the duality for a set of carefully chosen ρ_i 's in which the algebra is greatly simplified.

We choose

$$W_{\text{el}} = \sum_{i=1}^{F_X} \phi^{k_i-1} \text{Tr} \left[\frac{X_i^{k_i+1}}{k_i+1} \right] + \sum_{i=1}^{F_X-1} \text{Tr} [X_i X_{i+1}] \quad (3.43)$$

which gives the global symmetry group

$$\text{SU}(F_Q)_L \times \text{SU}(F_Q)_R \times \text{U}(1)_B \times \text{U}(1)_R. \quad (3.44)$$

and F -terms

$$\begin{aligned} \phi^{k_1-1} X_1^{k_1} + X_2 &= 0 \\ \phi^{k_i-1} X_i^{k_i} + X_{i-1} + X_{i+1} &= 0, \quad i \neq 1, F_X \\ \phi^{k_{F_X}-1} X_{F_X}^{k_{F_X}} + X_{F_X-1} &= 0. \end{aligned} \quad (3.45)$$

One can use the first of these equations to write X_2 in terms of X_1 . The second equation with $i = 2$ then allows X_3 to be expressed in terms of X_1 . One continues, using the second equation for increasing values of i , until all of the X_i have been expressed in terms of X_1 alone. The third equation then reduces down to

$$X_1^{k^*} \sim \mathcal{O} \left(X_1^{k^* - (k^*/k_{F_X}) + (k^*/k_{F_X} k_{F_X-1} k_{F_X-2})} \right) \quad (3.46)$$

where

$$k^* = \prod_{i=1}^{F_X} k_i. \quad (3.47)$$

The chiral ring is thus truncated at $X_1^{k^*}$.

Using X_1 as the active adjoint we now follow the usual procedure and use the dual gauge group $\text{SU}(n)$ with

$$n = k^* F_Q - N. \quad (3.48)$$

The mesons are

$$M_j = \tilde{Q} (\phi^{-1} X)^j Q, \quad j = 0, \dots, k^* - 1 \quad (3.49)$$

and the magnetic superpotential is

$$W_{\text{mag}} = \sum_{i=1}^{F_X} \phi^{k_i-1} \text{Tr} \left[\frac{x_i^{k_i+1}}{k_i+1} \right] + \sum_{i=1}^{F_X-1} \text{Tr} [x_i x_{i+1}] + \sum_{j=0}^{k^*-1} M_j \tilde{q} (\phi^{-1} x_1)^{k^*-1-j} q. \quad (3.50)$$

We summarise the matter content for the electric and magnetic theories in Tables 3.6 and 3.7 respectively. All $\text{U}(1)$ charge assignments can be seen to be consistent with both the meson structure and the magnetic superpotential as required.

The 't Hooft anomaly matching is straightforward, thanks to the simple R -charges of the adjoints. Indeed, the adjoints do not contribute to any of the anomalies we need to calculate as their fermionic components are not charged under any of the global symmetries. We find non-zero mixed anomalies

$$\begin{aligned} \text{U}(1)_B \times \text{SU}(F_Q)_L^2 &: 1 \\ \text{U}(1)_B \times \text{SU}(F_Q)_R^2 &: -1 \\ \text{U}(1)_R \times \text{SU}(F_Q)_L^2 &: -N^2/F_Q \\ \text{U}(1)_R \times \text{SU}(F_Q)_R^2 &: -N^2/F_Q \\ \text{U}(1)_R \times \text{U}(1)_B^2 &: -2 \\ \text{U}(1)_R &: -3 - N^2. \end{aligned} \quad (3.51)$$

	$SU(N)$	$SU(F_Q)_L$	$SU(F_Q)_R$	$U(1)_B$	$U(1)_R$
Q	\mathbf{N}	$\mathbf{F_Q}$	$\mathbf{1}$	$\frac{1}{N}$	$1 - \frac{N}{F_Q}$
\tilde{Q}	$\bar{\mathbf{N}}$	$\mathbf{1}$	$\bar{\mathbf{F_Q}}$	$-\frac{1}{N}$	$1 - \frac{N}{F_Q}$
X_i	\mathbf{adj}	$\mathbf{1}$	$\mathbf{1}$	0	1
ϕ	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	-1

Table 3.6: *The matter content of the electric theory in the F_X adjoint model.*

	$SU(n)$	$SU(F_Q)_L$	$SU(F_Q)_R$	$U(1)_B$	$U(1)_R$
q	\mathbf{n}	$\bar{\mathbf{F_Q}}$	$\mathbf{1}$	$\frac{1}{n}$	$1 - \frac{n}{F_Q}$
\tilde{q}	$\bar{\mathbf{n}}$	$\mathbf{1}$	$\mathbf{F_Q}$	$-\frac{1}{n}$	$1 - \frac{n}{F_Q}$
x_i	\mathbf{adj}	$\mathbf{1}$	$\mathbf{1}$	0	1
ϕ	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	-1
M_j	$\mathbf{1}$	$\mathbf{F_Q}$	$\bar{\mathbf{F_Q}}$	0	$2 \left(1 + j - \frac{N}{F_Q} \right)$

Table 3.7: *The matter content of the magnetic theory in the F_X adjoint model, $n = k^* F_Q - N$.*

and non-zero cubic anomalies

$$\begin{aligned}
SU(F_Q)_L^3 &: N \\
SU(F_Q)_R^3 &: -N \\
U(1)_R^3 &: -9 + N^2 - 2N^4/F_Q^2.
\end{aligned} \tag{3.52}$$

All anomalies are the same in both theories. Furthermore, the baryon matching goes exactly as in §3.1.1.

We used particular values for the ρ 's throughout this section but we expect the duality will still exist if we change these values. In general one is able to choose values for two of the ρ 's freely and still obtain a non-anomalous R -symmetry consistent with the superpotential. 't Hooft anomaly matching can impose extra constraints on the two free ρ 's. From considerations of theories with more than three adjoints, it appears that, for even F_X , there are no extra constraints. However, for odd F_X only one of these two ρ 's remains free. The second 'free' ρ is expressed in terms of the first, there being three possible solutions.

This is the one of two significant differences between theories with odd and even F_X . The second being the truncation of the chiral ring for $\phi = 0$. When $\phi = 0$ the chiral ring truncates trivially if F_X is even but does not truncate at all (with these particular values of the ρ 's) if F_X is odd. To see this one must notice that the second equation of Eq.(3.45) for $\phi = 0$ connects oddly/evenly labelled X_i 's to oddly/evenly labelled X_i 's. The first equation always gives $X_2 = 0$ at $\phi = 0$. One can then use the second equation to set $X_i = 0$ for all evenly labelled X_i . The third equation always gives $X_{F_X-1} = 0$ at $\phi = 0$. If F_X is even, $F_X - 1$ is odd and one can use the second equation to set $X_i = 0$ for all oddly labelled X_i as well. However, if F_X is odd we have no way of truncating the oddly labelled X_i . We expect that other choices of the ρ 's will allow a truncation to the deformed KSS model as in the 3-adjoint example discussed earlier.

4 RG flow

It is interesting to investigate the renormalisation group (RG) flow of some of the models we have discussed. We will use the a -maximisation theorem developed in Refs. [18, 19, 16, 17] to do this.

This theorem was used in the context of the KSS models in Refs. [20, 21]. In a nutshell, it tells us that the exact superconformal R -symmetry maximises the central charge

$$a \propto 3 \operatorname{Tr} [R^3] - \operatorname{Tr} [R] \quad (4.1)$$

where the trace is taken over all of the fermions in the theory. The R -symmetry used in Eq.(4.1) is the most general, non-anomalous R -symmetry in the theory which commutes with charge conjugation. Furthermore, the value of a should decrease as the theory flows into the infrared.

Using this theorem, our general approach will be thus:

1. Start at $W = 0$.
2. Use the a -maximisation theorem to find the exact superconformal R -symmetry.
3. Write down all relevant deformations to the superpotential ΔW consistent with the global symmetries. These are the gauge invariant operators with dimension < 3 , i.e. R -charge < 2 .
4. Determine which of these deformations independently lead to consistent, non-trivial fixed points. Consistency can only be achieved if $R_\phi \geq 2/3$ at the proposed fixed point, with the inequality saturated if and only if ϕ is a free field and therefore absent from the superpotential.
5. Repeat steps 3 and 4 about all consistent fixed points.

For step 4 we are using the fact that, if $R_\phi < 2/3$ at a fixed point, the singlet violates the unitarity bound; it is a gauge invariant spin-0 operator with dimension < 1 . According to Ref.[6] the solution to this problem is that ϕ is actually a free field charged under some accidental $U(1)$ symmetry. This accidental $U(1)$ charge mixes with the existing singlet R -charge to set $R_\phi = 2/3$. However, if the deformation we found in step 3 contained any powers of ϕ we arrive at a contradiction. A free field cannot appear in the superpotential, and we conclude that such a fixed point cannot exist.

4.1 Deformed KSS

Consider the deformed KSS model of §2. In the absence of the superpotential, the general, non-anomalous R -symmetry is

$$R_Q = R_{\tilde{Q}} = 1 - \hat{N} R_X \quad \text{where} \quad \hat{N} = \frac{N}{F_Q}, \quad (4.2)$$

and R_X and R_ϕ are arbitrary. Approximating the $SU(N)$ gauge group by a $U(N)$ gauge group and maximising a , one finds

$$R_X = \frac{10}{9 + 3\sqrt{20\hat{N}^2 - 1}}, \quad R_\phi = \frac{2}{3} \quad (4.3)$$

provided $\hat{N}^2 > 1/2$. When looking at the relevance of operators we will only be interested in the large \hat{N} limit. This is because any operator that is only relevant for sufficiently large \hat{N} will actually be dangerously irrelevant for smaller \hat{N} : quark mass terms allow one to integrate out quarks, reduce F_Q and therefore increase \hat{N} until it is large enough for the operator in question to become relevant. Note that working in the large \hat{N} limit also legitimizes the approximation of using a $U(N)$ gauge group, which greatly simplifies the algebra.

As $\hat{N} \rightarrow \infty$ in Eq.(4.3), $R_X \rightarrow 0$ and one finds several relevant deformations

$$\Delta W_0 = \lambda_0 X^{k_0+1}, \quad \Delta W_1 = \lambda_1 \phi X^{k_1+1} \quad \text{and} \quad \Delta W_2 = \lambda_2 \phi^2 X^{k_2+1}. \quad (4.4)$$

There are other deformations not involving the adjoints, but these are of less interest so we will not discuss them. We focus first on $W = \Delta W_2$, which fixes $R_\phi = 1 - (k_2 + 1) R_X/2$. a -maximisation

results in R -charges of $R_X = 0$ and $R_\phi = 1$ for large \hat{N} . $R_\phi > 2/3$ so the fixed point is consistent and should exist. Furthermore, one finds ΔW_0 and ΔW_1 are still relevant operators. We thus try $W = \Delta W_2 + \Delta W_1$.

With two terms in the superpotential the R -symmetry is completely fixed, with

$$R_X = \frac{2}{2k_1 - k_2 + 1} \quad \text{and} \quad R_\phi = 2 - \frac{2(k_1 + 1)}{2k_1 - k_2 + 1}. \quad (4.5)$$

There are gauge singlets appearing in both terms so, going back to §2, this is the $\rho_0 \neq 0$, $\rho_l \neq 0$ case. If one wants a duality, Table 2.4 tells us that we require either

$$1 = 2 \frac{k_1 - 1}{k_2 - 1} \quad \text{or} \quad 2 = \frac{k_1 - 1}{k_2 - 1} \quad (4.6)$$

depending on whether $k_2 > k_1$ (and $\rho_0 = 2$, $\rho_l = 1$) or $k_2 < k_1$ (and $\rho_0 = 1$, $\rho_l = 2$). Neither of these choices result in $R_\phi > 2/3$ for non-zero values of both k_2 and k_1 . We conclude that a non-trivial fixed point for $W = \Delta W_2 + \Delta W_1$ cannot exist in this class of electric/magnetic duals.

Instead, we try $W = \Delta W_2 + \Delta W_0$. Here one finds the fixed R -charges

$$R_X = \frac{2}{k_0 + 1} \quad \text{and} \quad R_\phi = \frac{k_0 - k_2}{k_0 + 1}. \quad (4.7)$$

This superpotential falls into the $\rho_0 = 0$, $\rho_l \neq 0$ category for which a duality is always possible as long as $k_0 > k_2$. One can easily show that, for $k_0 > 3k_2 + 2$, $R_\phi > 2/3$. A non-trivial fixed point for $W = \Delta W_2 + \Delta W_0$ is therefore a consistent proposition.

We have continued this analysis for all combinations of the deformations Eq.(4.4) maintaining the duality. The results are summarised in Table 4.1 and the corresponding RG flow with the maximum number of fixed points is sketched in Figure 4.1. The flow along the dashed line in

W	$+\Delta W_2$		$+\Delta W_1$		$+\Delta W_0$	
	Relevant	Fixed Pt	Relevant	Fixed Pt	Relevant	Fixed Pt
0	yes	yes	yes	yes	yes	yes
ΔW_2	—	—	yes	no	yes	$k_0 > 3k_2 + 2$
ΔW_1	no	no	—	—	yes	$k_0 > \frac{3}{2}k_1 + \frac{1}{2}$
ΔW_0	$k_2 < \frac{1}{3}k_0 + \frac{1}{3}$	$k_2 < \frac{1}{3}k_0 - \frac{2}{3}$	$k_1 < \frac{2}{3}k_0 + \frac{2}{3}$	$k_1 < \frac{2}{3}k_0 - \frac{1}{3}$	—	—

Table 4.1: *The fixed point analysis for the deformed KSS model of §2 with the deformations Eq.(4.4) allowed by the duality.*

Figure 4.1 (connecting the λ_2 - λ_0 and λ_1 - λ_0 fixed points) depends on the values of k_2 and k_1 . The superpotential along this line is $W = \Delta W_2 + \Delta W_1 + \Delta W_0$, which only admits an R -symmetry if $k_2 = (k_1 + k_0)/2$. In this case the flow along the dashed line preserves a , suggesting a line of conformal fixed points. If this is true we can use the reasoning of Ref.[22] to argue that the duality is *exact* along the dashed line, rather than just an infrared duality. When $k_2 \neq (k_1 + k_0)/2$ there is no R -symmetry along the dashed line. The difference in a between the λ_2 - λ_0 and λ_1 - λ_0 fixed points is then

$$\begin{aligned} \Delta a &= a(\lambda_2 - \lambda_0) - a(\lambda_1 - \lambda_0) \\ &= \frac{2k_1 - (k_2 + k_0)}{(1 + k_0)^3} (8 - 11k_0 + 2k_0^2 + 18k_1 - 12k_0k_1 + 12k_1^2 + 9k_2 - 3k_0k_2 + 6k_1k_2 + 3k_2^2). \end{aligned} \quad (4.8)$$

For $\Delta a > 0$ the flow is from λ_2 - λ_0 to λ_1 - λ_0 . If $\Delta a < 0$ the flow is from λ_1 - λ_0 to λ_2 - λ_0

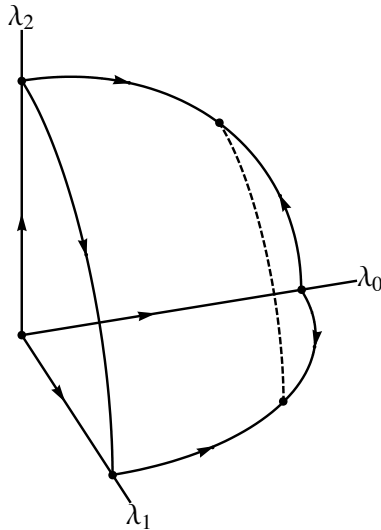


Figure 4.1: The RG flow for the deformed KSS model of §2 with the deformations Eq.(4.4) allowed by the duality. $k_0 > 3k_2 + 2$ and $k_0 > \frac{3}{2}k_1 + \frac{1}{2}$ to ensure the λ_2 - λ_0 and λ_1 - λ_0 fixed points exist. The flow along the dashed line connecting the two fixed points depends on the values of k_2 and k_1 .

4.2 More adjoints

In models with two adjoints it was shown in Ref.[23] that all operators of order less than four in the adjoints are relevant around the $W = 0$ fixed point. The addition of gauge singlets does not change this result. The only such operator appearing in the models of §3.1 is the mass term XY . If one considers deforming the superpotential to $W = \lambda XY$ the R -symmetry must satisfy $R_Y = 2 - R_X$ so the corresponding fermions have equal and opposite R -charges. The contribution of the adjoints to a is therefore completely cancelled, leaving the value of R_X undetermined. Despite this freedom the RG flow of the two adjoint model is not very interesting. Indeed, a quick glance at Table 3.1 shows that R_ϕ is always negative for non-trivial choices of superpotential. We thus conclude that the interesting terms in the two adjoint model are usually irrelevant and certainly don't generate non-trivial fixed points.

This is generally the case for models with an even number of adjoints. After the mass terms have been added, there will be one adjoint X_j for each adjoint X_i such that $R_{X_j} = 2 - R_{X_i}$. Consequently, none of the R_{X_i} will appear in a and we will end up with the situation where all interesting deformations to the superpotential are irrelevant. For models with an odd number of adjoints this is not the case. After adding the mass terms the R -charge cancellations will proceed as before but there will always be one adjoint whose R -charge cannot be paired up in this way. We will thus be left with a single, undetermined R_{X_i} appearing in a . The RG flow then proceeds similarly to the model with a single adjoint, i.e. the deformed KSS model. It should therefore come as no surprise that, when $\phi = 0$, the models with an even number of adjoints flow to SQCD, while the models with an odd number of adjoints appear to flow to the deformed KSS model.

Note that we are generally interested in the regions of moduli space where $\phi \neq 0$ and the R -symmetry is spontaneously broken. All of the constraints we have derived in this section use R -symmetry arguments so do not apply at these points, leaving much more freedom to deform the superpotential.

5 Models with antisymmetrics

Building on the success with adjoints in §3, we now turn to models containing antisymmetric representations of the gauge group. We will base these on the models of Ref.[14]. These models are similar in spirit to KS duality but have a more complicated meson and baryon structure. Once again, all electric theories are based on SQCD with gauge group $SU(N)$. The magnetic gauge group will be $SU(n)$ for some n to be determined.

5.1 Two antisymmetrics

We will start with two antisymmetrics A and B along with their conjugates \tilde{A} and \tilde{B} . The electric superpotential is

$$W_{\text{el}} = \phi^{\rho_A} \text{Tr} \left[\frac{(A\tilde{A})^{k_A+1}}{k_A+1} \right] + \phi^{\rho_B} \text{Tr} \left[\frac{(B\tilde{B})^{k_B+1}}{k_B+1} \right] + \phi^\sigma \left(\text{Tr} [A\tilde{B}] + \text{Tr} [\tilde{A}B] \right). \quad (5.1)$$

for some ρ_A , ρ_B and σ . The inclusion of $\sigma \neq 0$ turns out to be necessary for baryon matching; we will discuss it in more detail in §5.1.1. For now, we merely note that

$$\sigma = -\frac{\rho_A(1+k_B) + \rho_B(1+k_A)}{2k_A k_B}. \quad (5.2)$$

The non-anomalous, global symmetry group for arbitrary ρ_A , ρ_B and σ is

$$SU(F_Q)_L \times SU(F_Q)_R \times U(1)_B \times U(1)_A \times U(1)_R \quad (5.3)$$

where $U(1)_R$ is a unique R -symmetry.

The F -terms for the antisymmetrics give

$$\begin{aligned} \phi^{\rho_A} \tilde{A}(A\tilde{A})^{k_A} + \phi^\sigma \tilde{B} &= 0 \\ \phi^{\rho_A} (A\tilde{A})^{k_A} A + \phi^\sigma B &= 0 \\ \phi^{\rho_B} \tilde{B}(B\tilde{B})^{k_B} + \phi^\sigma \tilde{A} &= 0 \\ \phi^{\rho_B} (B\tilde{B})^{k_B} B + \phi^\sigma A &= 0. \end{aligned} \quad (5.4)$$

One uses the first two equations to write B and \tilde{B} in terms of A and \tilde{A} then, upon substitution into the last two equations, one finds

$$\begin{aligned} \phi^{(\rho_A-\sigma)(2k_B+1)-\sigma+\rho_B} \tilde{A}(A\tilde{A})^{2k_A k_B+k_A+k_B} &= \tilde{A} \\ \phi^{(\rho_A-\sigma)(2k_B+1)-\sigma+\rho_B} (A\tilde{A})^{2k_A k_B+k_A+k_B} A &= A. \end{aligned} \quad (5.5)$$

If $\phi \neq 0$ the chiral ring truncates at $\tilde{A}(A\tilde{A})^{2k_A k_B+k_A+k_B}$ and $(A\tilde{A})^{2k_A k_B+k_A+k_B} A$ or, equivalently with B and \tilde{B} . We will discuss the $\phi = 0$ case later.

Once again, Eq.(5.5) looks similar to the truncation equations of the base models Ref.[14] for $\phi \neq 0$. We therefore assume the duality proceeds in the same way, but using the effective value of k

$$k^* = \frac{1}{2} [(2k_A+1)(2k_B+1) - 1]. \quad (5.6)$$

This implies the meson structure

$$\begin{aligned} M_j &= \tilde{Q}(\phi^{2\alpha} A\tilde{A})^j Q, \quad j = 0, \dots, k^* \\ P_j &= Q[\phi^\alpha \tilde{A}(\phi^{2\alpha} A\tilde{A})^j] Q, \quad j = 0, \dots, k^* - 1 \\ \tilde{P}_j &= \tilde{Q}[(\phi^{2\alpha} A\tilde{A})^j \phi^\alpha A] \tilde{Q}, \quad j = 0, \dots, k^* - 1 \end{aligned} \quad (5.7)$$

where we have learnt from our experiences with adjoints and included a ϕ^α with each antisymmetric from the start. The dual gauge group is $SU(n)$ with

$$n = (2k^* + 1) F_Q - 4k^* - N. \quad (5.8)$$

With a magnetic superpotential

$$\begin{aligned}
W_{\text{mag}} = & \phi^{\rho_A} \text{Tr} \left[\frac{(a\tilde{a})^{k_A+1}}{k_A+1} \right] + \phi^{\rho_B} \text{Tr} \left[\frac{(b\tilde{b})^{k_B+1}}{k_B+1} \right] + \phi^\sigma \left(\text{Tr} [a\tilde{b}] + \text{Tr} [\tilde{a}b] \right) + \\
& \sum_{j=0}^{k^*-1} P_j q [\phi^\alpha \tilde{a} (\phi^{2\alpha} a \tilde{a})^{k^*-1-j}] q + \sum_{j=0}^{k^*-1} \tilde{P}_j \tilde{q} [(\phi^{2\alpha} a \tilde{a})^{k^*-1-j} \phi^\alpha a] \tilde{q} + \\
& \sum_{j=0}^{k^*} M_j \tilde{q} (\phi^{2\alpha} a \tilde{a})^{k^*-j} q
\end{aligned} \tag{5.9}$$

one finds the matter has the symmetry properties laid out in Table 5.1 for the electric theory and Table 5.2 for the magnetic theory. The value of α is determined by demanding that the meson terms in the magnetic superpotential have R -charge $+2$. This yields

$$\alpha = \frac{\rho_A (1 + k^*) + \rho_B (1 + 2k_A)}{2k_A k^*}. \tag{5.10}$$

	$\text{SU}(N)$	$\text{SU}(F_Q)_L$	$\text{SU}(F_Q)_R$	$\text{U}(1)_B$	$\text{U}(1)_A$	$\text{U}(1)_R$
Q	\mathbf{N}	\mathbf{F}_Q	$\mathbf{1}$	$\frac{1}{N}$	0	$1 - \frac{N+2k^*}{F_Q(1+k^*)}$
\tilde{Q}	$\overline{\mathbf{N}}$	$\mathbf{1}$	$\overline{\mathbf{F}_Q}$	$-\frac{1}{N}$	0	$1 - \frac{N+2k^*}{F_Q(1+k^*)}$
A	\mathbf{asym}	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{N}$	1	$\frac{\rho_A(1+2k_B+k_B k^*)+\rho_B(1+k^*)}{(1+k^*)[\rho_A(1+k_B)+\rho_B(1+k_A)]}$
\tilde{A}	$\overline{\mathbf{asym}}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{2}{N}$	-1	$\frac{\rho_A(1+2k_B+k_B k^*)+\rho_B(1+k^*)}{(1+k^*)[\rho_A(1+k_B)+\rho_B(1+k_A)]}$
B	\mathbf{asym}	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{N}$	1	$\frac{\rho_B(1+2k_A+k_A k^*)+\rho_A(1+k^*)}{(1+k^*)[\rho_A(1+k_B)+\rho_B(1+k_A)]}$
\tilde{B}	$\overline{\mathbf{asym}}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{2}{N}$	-1	$\frac{\rho_B(1+2k_A+k_A k^*)+\rho_A(1+k^*)}{(1+k^*)[\rho_A(1+k_B)+\rho_B(1+k_A)]}$
ϕ	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	$-\frac{2k_A k_B k^*}{(1+k^*)[\rho_A(1+k_B)+\rho_B(1+k_A)]}$

Table 5.1: *The matter content of the electric theory in the two antisymmetric model.*

Moving on to 't Hooft anomaly matching the picture is similar to §3. The non-zero mixed anomalies are

$$\begin{aligned}
\text{U}(1)_B \times \text{SU}(F_Q)_L^2 & : 1 \\
\text{U}(1)_B \times \text{SU}(F_Q)_R^2 & : -1 \\
\text{U}(1)_R \times \text{SU}(F_Q)_L^2 & : -N(N+2k^*)/F_Q(1+k^*) \\
\text{U}(1)_R \times \text{SU}(F_Q)_R^2 & : -N(N+2k^*)/F_Q(1+k^*) \\
\text{U}(1)_R \times \text{U}(1)_B^2 & : -2(1+2k_A)(1+2k_B)/(1+k^*) \\
\text{U}(1)_R \times \text{U}(1)_A^2 & : -k^*N(N-1)/(1+k^*) \\
\text{U}(1)_R & : -2 - \frac{3k^*N+N^2}{1+k^*} - \frac{2k_A k_B k^*}{(1+k^*)[\rho_A(1+k_B)+\rho_B(1+k_A)]}. \tag{5.11}
\end{aligned}$$

They are the same in both the electric and magnetic theories. The cubic flavour anomalies are

$$\begin{aligned}
\text{SU}(F_Q)_L^3 & : N \\
\text{SU}(F_Q)_R^3 & : -N
\end{aligned} \tag{5.12}$$

in both theories; the $\text{U}(1)_B^3$ and $\text{U}(1)_A^3$ anomalies are zero. As in the three adjoint model, the cubic R -symmetry anomalies are both very complicated expressions which do not immediately

	$SU(n)$	$SU(F_Q)_L$	$SU(F_Q)_R$	$U(1)_B$	$U(1)_A$	$U(1)_R$
q	\mathbf{n}	$\overline{\mathbf{F}_Q}$	$\mathbf{1}$	$\frac{1}{n}$	$\frac{(F_Q-2)k^*}{n}$	$1 - \frac{n+2k^*}{F_Q(1+k^*)}$
\tilde{q}	$\overline{\mathbf{n}}$	$\mathbf{1}$	\mathbf{F}_Q	$-\frac{1}{n}$	$-\frac{(F_Q-2)k^*}{n}$	$1 - \frac{n+2k^*}{F_Q(1+k^*)}$
a	\mathbf{asym}	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{n}$	$\frac{N-F_Q}{n}$	$\frac{\rho_A(1+2k_B+k_Bk^*)+\rho_B(1+k^*)}{(1+k^*)[\rho_A(1+k_B)+\rho_B(1+k_A)]}$
\tilde{a}	$\overline{\mathbf{asym}}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{2}{n}$	$-\frac{N-F_Q}{n}$	$\frac{\rho_A(1+2k_B+k_Bk^*)+\rho_B(1+k^*)}{(1+k^*)[\rho_A(1+k_B)+\rho_B(1+k_A)]}$
b	\mathbf{asym}	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{n}$	$\frac{N-F_Q}{n}$	$\frac{\rho_B(1+2k_A+k_Ak^*)+\rho_A(1+k^*)}{(1+k^*)[\rho_A(1+k_B)+\rho_B(1+k_A)]}$
\tilde{b}	$\overline{\mathbf{asym}}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{2}{n}$	$-\frac{N-F_Q}{n}$	$\frac{\rho_B(1+2k_A+k_Ak^*)+\rho_A(1+k^*)}{(1+k^*)[\rho_A(1+k_B)+\rho_B(1+k_A)]}$
ϕ	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	$-\frac{2k_Ak_Bk^*}{(1+k^*)[\rho_A(1+k_B)+\rho_B(1+k_A)]}$
M_j	$\mathbf{1}$	\mathbf{F}_Q	$\overline{\mathbf{F}_Q}$	0	0	$\frac{F_Q(2+2j+2k^*)-2N-4k^*}{F_Q(1+k^*)}$
P_j	$\mathbf{1}$	\mathbf{asym}	$\mathbf{1}$	0	-1	$\frac{F_Q(3+2j+2k^*)-2N-4k^*}{F_Q(1+k^*)}$
\tilde{P}_j	$\mathbf{1}$	$\mathbf{1}$	$\overline{\mathbf{asym}}$	0	1	$\frac{F_Q(3+2j+2k^*)-2N-4k^*}{F_Q(1+k^*)}$

Table 5.2: *The matter content of the magnetic theory in the two antisymmetric model, $n = (2k^* + 1)F_Q - 4k^* - N$.*

match. One can write the difference as

$$f_{2A}(\rho, k) [\rho_B(1+2k_A) + \rho_A(1+k^*)] [\rho_B(1+k^*) + \rho_A(1+2k_B)] \quad (5.13)$$

where

$$f_{2A}(\rho, k) = -\frac{3k_Ak_Bk^* [F_Q(1+2k^*) - 1 - 4k^*] [F_Q(1+2k^*) - 2N - 4k^*]}{(1+k^*)^3 [\rho_A(1+k_B) + \rho_B(1+k_A)]^2}. \quad (5.14)$$

When Eq.(5.13) is equal to zero all cubic anomalies match. Solving for ρ_B yields two possibilities;

$$\rho_B = -\frac{k^* - k_B}{k_A(1+k^*)}\rho_A \quad \text{or} \quad \rho_B = -\frac{1+k^*}{1+2k_A}\rho_A. \quad (5.15)$$

Note that $\rho_A = \rho_B = 0$ is not allowed as the R -charge of the singlet is not well defined. Substituting into Eq.(5.2) one finds

$$\sigma = -\frac{k^*}{2k_A(1+k^*)}\rho_A \quad \text{or} \quad \sigma = \frac{k^*}{2k_B(1+2k_A)}\rho_A \quad (5.16)$$

respectively.

For $\phi = 0$ there are two truncation possibilities. Choosing $\rho_A > 0$ one finds strictly negative values of ρ_B from Eq.(5.15). With this in mind, the $\sigma > 0$ solution of Eq.(5.16) reduces the F -term equations Eq.(5.4) to

$$\tilde{B}(B\tilde{B})^{k_B} = (B\tilde{B})^{k_B}B = 0 \quad (5.17)$$

at $\phi = 0$ and the chiral ring does not truncate for A and \tilde{A} . On the other hand, the chiral ring truncates trivially for the $\sigma < 0$ solution of Eq.(5.16). The first two equations force $B = \tilde{B} = 0$ which, in conjunction with the last two equations, forces $A = \tilde{A} = 0$. If $\rho_A < 0$ one finds $\rho_B > 0$ and the situation is identical, but with A 's and B 's exchanged.

5.1.1 Baryon matching

The baryon matching scheme of Ref.[14], in conjunction with our meson structure Eq.(5.7) and Eq.(5.10), suggest electric baryons of the form

$$B_r = (\phi^\alpha A)^r Q^{N-2r} \quad (5.18)$$

and magnetic baryons

$$b_r = (\phi^\alpha a)^{(F_Q-2)k^*-r} q^{F_Q-N+2r} \quad (5.19)$$

with the baryon map $B_r \leftrightarrow b_r$. This map is totally consistent with all global symmetries. The powers of the singlets are precisely those needed to match the R -charges of the baryons. However, this is only the case for the value of σ set down in Eq.(5.2).

If we had left σ as an undetermined parameter we would have found

$$\alpha = \frac{\rho_A (4 - F_Q + 4k_B) + \rho_B (4 - F_Q + 4k_A) - 2\rho_B k_A F_Q - 2\sigma k_B (F_Q + 2k_A F_Q - 4K_A)}{2F_Q k^*} \quad (5.20)$$

when we constrained the meson terms in the magnetic superpotential to have R -charge +2. Using this value of α in the baryon map, the R -charges do not generally match up. Even worse, the difference is dependent on the value of r used in the baryons. Hence we are not free to choose σ arbitrarily. If we wish to construct a baryon map which satisfies R -charge matching independently of r we are forced to choose σ as in Eq.(5.2).

5.2 Three antisymmetrics

We now add an extra antisymmetric C and its conjugate \tilde{C} to the electric theory of §5.1, leading to a superpotential

$$W_{\text{el}} = \phi^{\rho_A} \text{Tr} \left[\frac{(A\tilde{A})^{k_A+1}}{k_A+1} \right] + \phi^{\rho_B} \text{Tr} \left[\frac{(B\tilde{B})^{k_B+1}}{k_B+1} \right] + \phi^{\rho_C} \text{Tr} \left[\frac{(C\tilde{C})^{k_C+1}}{k_C+1} \right] + \phi^\sigma \left(\text{Tr} [A\tilde{B}] + \text{Tr} [\tilde{A}B] + \text{Tr} [B\tilde{C}] + \text{Tr} [\tilde{B}C] \right). \quad (5.21)$$

We define

$$k^* = \frac{1}{2} [(2k_A+1)(2k_B+1)(2k_C+1) - 1] \quad (5.22)$$

at this point to simplify the following expressions. Note that, in line with our previous experiences, this is the value of k^* we expect. For arbitrary values of ρ_B , the electric theory has no non-anomalous R -symmetry. One must choose

$$\rho_B = \frac{\rho_C (k_A + k_B + 2k_A k_B) - \rho_A (k_B + k_C + 2k_B k_C)}{k_C - k_A} + 2(1 + k_B)\sigma \quad (5.23)$$

in order for the theory to possess one. Furthermore, one requires

$$\sigma = -\frac{\rho_C (k^* - k_A) - \rho_A (k^* - k_C)}{2(1 + k^*)(k_C - k_A)} \quad (5.24)$$

for the baryon map to work out, as discussed in §5.1.1. With ρ_B and σ fixed, the non-anomalous, global symmetry group is

$$\text{SU}(F_Q)_L \times \text{SU}(F_Q)_R \times \text{U}(1)_B \times \text{U}(1)_A \times \text{U}(1)_R \quad (5.25)$$

as it was for the two antisymmetric model.

The F -terms for the antisymmetrics give

$$\begin{aligned} \phi^{\rho_A} \tilde{A}(A\tilde{A})^{k_A} + \phi^\sigma \tilde{B} &= 0 \\ \phi^{\rho_A} (A\tilde{A})^{k_A} A + \phi^\sigma B &= 0 \\ \phi^{\rho_B} \tilde{B}(B\tilde{B})^{k_B} + \phi^\sigma (\tilde{A} + \tilde{C}) &= 0 \\ \phi^{\rho_B} (B\tilde{B})^{k_B} B + \phi^\sigma (A + C) &= 0 \\ \phi^{\rho_C} \tilde{C}(C\tilde{C})^{k_C} + \phi^\sigma \tilde{B} &= 0 \\ \phi^{\rho_C} (C\tilde{C})^{k_C} C + \phi^\sigma B &= 0. \end{aligned} \quad (5.26)$$

One can use the first two equations to write B and \tilde{B} in terms of A and \tilde{A} , then the middle two equations to write C and \tilde{C} in terms of A and \tilde{A} . Finally, the last two equations can be employed to find

$$\begin{aligned}\tilde{A}(A\tilde{A})^{k^*} &\sim O\left(\tilde{A}(A\tilde{A})^{k^*-2k_Ak_B-k_A-k_B}\right) \\ (A\tilde{A})^{k^*}A &\sim O\left((A\tilde{A})^{k^*-2k_Ak_B-k_A-k_B}A\right).\end{aligned}\quad (5.27)$$

As long as $\phi \neq 0$ the chiral ring is truncated at $\tilde{A}(A\tilde{A})^{k^*}$ and $(A\tilde{A})^{k^*}A$. Equivalently, we could truncate in exactly the same way using C and \tilde{C} . The $\phi = 0$ case is discussed later.

Assuming $\phi \neq 0$ and using A as the active antisymmetric (the argument would be equivalent if we used C instead) we proceed as usual. The dual gauge group is $SU(n)$ with

$$n = (2k^* + 1)F_Q - 4k^* - N. \quad (5.28)$$

k^* is defined is Eq.(5.22). The mesons are

$$\begin{aligned}M_j &= \tilde{Q}(\phi^{2\alpha}A\tilde{A})^jQ, \quad j = 0, \dots, k^* \\ P_j &= Q[\phi^\alpha\tilde{A}(\phi^{2\alpha}A\tilde{A})^j]Q, \quad j = 0, \dots, k^* - 1 \\ \tilde{P}_j &= \tilde{Q}[(\phi^{2\alpha}A\tilde{A})^j\phi^\alpha A]\tilde{Q}, \quad j = 0, \dots, k^* - 1\end{aligned}\quad (5.29)$$

with

$$\alpha = \frac{\rho_C(k^* - k_A) - \rho_A(k^* - k_C)}{2(1 + k^*)(k_C - k_A)} = -\sigma. \quad (5.30)$$

The magnetic superpotential is

$$\begin{aligned}W_{\text{el}} &= \phi^{\rho_A} \text{Tr} \left[\frac{(a\tilde{a})^{k_A+1}}{k_A+1} \right] + \phi^{\rho_B} \text{Tr} \left[\frac{(b\tilde{b})^{k_B+1}}{k_B+1} \right] + \phi^{\rho_C} \text{Tr} \left[\frac{(c\tilde{c})^{k_C+1}}{k_C+1} \right] + \\ &\phi^\sigma \left(\text{Tr} [a\tilde{b}] + \text{Tr} [\tilde{a}b] + \text{Tr} [b\tilde{c}] + \text{Tr} [\tilde{b}c] \right) + \\ &\sum_{j=0}^{k^*-1} P_j q [\phi^\alpha \tilde{a} (\phi^{2\alpha} a \tilde{a})^{k^*-1-j}] q + \sum_{j=0}^{k^*-1} \tilde{P}_j \tilde{q} [(\phi^{2\alpha} a \tilde{a})^{k^*-1-j} \phi^\alpha a] \tilde{q} + \\ &\sum_{j=0}^{k^*} M_j \tilde{q} (\phi^{2\alpha} a \tilde{a})^{k^*-j} q.\end{aligned}\quad (5.31)$$

We summarise the matter content for the electric and magnetic theories in Tables 5.3 and 5.4 respectively. All $U(1)$ charge assignments are consistent with both the meson structure and the magnetic superpotential as required.

We now consider the 't Hooft anomaly matching. The mixed anomalies are exactly the same as those of the two antisymmetric model Eq.(5.11), but with the new version of k^* given by Eq.(5.22) and

$$U(1)_R : -2 - \frac{3k^*N + N^2}{1 + k^*} - \frac{2(k_C - k_A)}{\rho_C(1 + k_A) - \rho_A(1 + k_C)}. \quad (5.32)$$

The cubic anomalies are also equivalent with the exception of the $U(1)_R^3$ anomaly; the flavour anomalies are

$$\begin{aligned}SU(F_Q)_L^3 &: N \\ SU(F_Q)_R^3 &: -N\end{aligned}\quad (5.33)$$

in both theories; the $U(1)_B^3$ and $U(1)_A^3$ anomalies are zero. Again, the cubic R -symmetry anomalies do not immediately match. One can write the difference as

$$f_{3A}(\rho, k) [\rho_C k_A - \rho_A k_C] [\rho_C(k^* - k_A) - \rho_A(k^* - k_C)]^2 \quad (5.34)$$

	$SU(N)$	$SU(F_Q)_L$	$SU(F_Q)_R$	$U(1)_B$	$U(1)_A$	$U(1)_R$
Q	\mathbf{N}	\mathbf{F}_Q	$\mathbf{1}$	$\frac{1}{N}$	0	$1 - \frac{N+2k^*}{F_Q(1+k^*)}$
\tilde{Q}	$\overline{\mathbf{N}}$	$\mathbf{1}$	$\overline{\mathbf{F}_Q}$	$-\frac{1}{N}$	0	$1 - \frac{N+2k^*}{F_Q(1+k^*)}$
A	\mathbf{asym}	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{N}$	1	$\frac{\rho_C - \rho_A}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
\tilde{A}	$\overline{\mathbf{asym}}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{2}{N}$	-1	$\frac{\rho_C - \rho_A}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
B	\mathbf{asym}	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{N}$	1	$\frac{1}{1+k^*} + \frac{2(\rho_C k_A - \rho_A k_C)}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
\tilde{B}	$\overline{\mathbf{asym}}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{2}{N}$	-1	$\frac{1}{1+k^*} + \frac{2(\rho_C k_A - \rho_A k_C)}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
C	\mathbf{asym}	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{N}$	1	$\frac{\rho_C - \rho_A}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
\tilde{C}	$\overline{\mathbf{asym}}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{2}{N}$	-1	$\frac{\rho_C - \rho_A}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
ϕ	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	$-\frac{2(k_C - k_A)}{\rho_C(1+k_A) - \rho_A(1+k_C)}$

Table 5.3: *The matter content of the electric theory in the three antisymmetric model.*

	$SU(n)$	$SU(F_Q)_L$	$SU(F_Q)_R$	$U(1)_B$	$U(1)_A$	$U(1)_R$
q	\mathbf{n}	$\overline{\mathbf{F}_Q}$	$\mathbf{1}$	$\frac{1}{n}$	$\frac{(F_Q-2)k^*}{n}$	$1 - \frac{n+2k^*}{F_Q(1+k^*)}$
\tilde{q}	$\overline{\mathbf{n}}$	$\mathbf{1}$	\mathbf{F}_Q	$-\frac{1}{n}$	$-\frac{(F_Q-2)k^*}{n}$	$1 - \frac{n+2k^*}{F_Q(1+k^*)}$
a	\mathbf{asym}	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{n}$	$\frac{N-F_Q}{n}$	$\frac{\rho_C - \rho_A}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
\tilde{a}	$\overline{\mathbf{asym}}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{2}{n}$	$-\frac{N-F_Q}{n}$	$\frac{\rho_C - \rho_A}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
b	\mathbf{asym}	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{n}$	$\frac{N-F_Q}{n}$	$\frac{1}{1+k^*} + \frac{2(\rho_C k_A - \rho_A k_C)}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
\tilde{b}	$\overline{\mathbf{asym}}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{2}{n}$	$-\frac{N-F_Q}{n}$	$\frac{1}{1+k^*} + \frac{2(\rho_C k_A - \rho_A k_C)}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
c	\mathbf{asym}	$\mathbf{1}$	$\mathbf{1}$	$\frac{2}{n}$	$\frac{N-F_Q}{n}$	$\frac{\rho_C - \rho_A}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
\tilde{c}	$\overline{\mathbf{asym}}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{2}{n}$	$-\frac{N-F_Q}{n}$	$\frac{\rho_C - \rho_A}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
ϕ	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	$-\frac{2(k_C - k_A)}{\rho_C(1+k_A) - \rho_A(1+k_C)}$
M_j	$\mathbf{1}$	\mathbf{F}_Q	$\overline{\mathbf{F}_Q}$	0	0	$\frac{F_Q(2+2j+2k^*)-2N-4k^*}{F_Q(1+k^*)}$
P_j	$\mathbf{1}$	\mathbf{asym}	$\mathbf{1}$	0	-1	$\frac{F_Q(3+2j+2k^*)-2N-4k^*}{F_Q(1+k^*)}$
\tilde{P}_j	$\mathbf{1}$	$\mathbf{1}$	$\overline{\mathbf{asym}}$	0	1	$\frac{F_Q(3+2j+2k^*)-2N-4k^*}{F_Q(1+k^*)}$

Table 5.4: *The matter content of the magnetic theory in the three antisymmetric model, $n = (2k^* + 1)F_Q - 4k^* - N$.*

where

$$f_{3A}(\rho, k) = -\frac{6[F_Q(1+2k^*) - 1 - 4k^*][F_Q(1+2k^*) - 2N - 4k^*]}{(1+k^*)^2[\rho_C(1+k_A) - \rho_A(1+k_C)]^3}. \quad (5.35)$$

Where Eq.(5.34) is equal to zero all cubic anomalies match. Solving for ρ_C yields two possibilities;

$$\rho_C = \frac{k_C}{k_A} \rho_A \quad \text{or} \quad \rho_C = \frac{k^* - k_C}{k^* - k_A} \rho_A \quad (5.36)$$

which, using Eq.(5.23), result in

$$\rho_B = -\frac{k^* - k_B}{k_A(1+k^*)} \rho_A \quad \text{or} \quad \rho_B = -\frac{k_B + 2k_B k^* + k^*}{k^* - k_A} \rho_A \quad (5.37)$$

respectively. Any choice of the ρ s satisfying both Eq.(5.36), Eq.(5.37) and giving well defined R -charges is allowed. Note that demanding a well defined R -charge for the singlet once again eliminates the solution $\rho_A = \rho_B = \rho_C = 0$. Substituting into Eq.(5.24) and Eq.(5.30) one finds

$$\sigma = -\alpha = -\frac{k^*}{2k_A(1+k^*)}\rho_A \quad \text{or} \quad \sigma = -\alpha = 0 \quad (5.38)$$

respectively.

As another check, we consider the baryon matching. This goes exactly as in §5.1.1 for all allowed choices of ρ_A .

When $\phi = 0$, we get a deformed version of the models in Ref.[14]. Explicitly, Eq.(5.36) and Eq.(5.37) show that ρ_A and ρ_C always have the same sign whereas ρ_B always has the opposite sign. σ either has the opposite sign to ρ_A and ρ_C or is zero. We must therefore consider the F -term equations Eq.(5.26) for several possibilities. If $\rho_A, \rho_C > 0$ then $\rho_B < 0$ and $\sigma \leq 0$. Eq.(5.26) tells us that $B = \tilde{B} = 0$ at $\phi = 0$ but says nothing about A or C , hence the chiral ring is not truncated. If $\rho_A, \rho_C < 0$ then $\rho_B > 0$ and $\sigma \geq 0$. The middle two equations reduce down to $A = -C$ and $\tilde{A} = -\tilde{C}$ in this case. The remaining equations of Eq.(5.26) then only have finite field value solutions at $\phi = 0$ if $\tilde{A}(A\tilde{A})^{k_A} = (A\tilde{A})^{k_A}A = 0$ and $\tilde{A}(A\tilde{A})^{k_C} = (A\tilde{A})^{k_C}A = 0$. If $\sigma = 0$ this solution also sets $B = \tilde{B} = 0$ and the chiral ring is truncated at the lowest of k_A and k_C ; we are, as claimed, left with a deformed version of the models in Ref.[14]. However, if $\sigma > 0$ B does not appear in the F -term equations at $\phi = 0$ and the chiral ring remains untruncated.

5.3 F_A antisymmetrics?

The similarities between the first of the solutions for ρ_B and σ in the two and three antisymmetric models (see Eq.(5.15), Eq.(5.16) and Eq.(5.37), Eq.(5.38) respectively) suggests a pattern which may be extended to an arbitrary number of antisymmetrics. This was certainly the case in §3.3 for models with adjoints. Thus far, we have been unable to realise a duality for a model with F_A antisymmetrics.

We believe such a duality does exist. Indeed, if the electric theory for such a model were to have superpotential

$$W_{\text{el}} = \sum_{i=1}^{F_A} \phi^{\rho_i} \text{Tr} \left[\frac{(A_i \tilde{A}_i)^{k_i+1}}{k_i+1} \right] + \phi^\sigma \sum_{i=1}^{F_A-1} \left(\text{Tr} [A_i \tilde{A}_{i+1}] + \text{Tr} [\tilde{A}_i A_{i+1}] \right) \quad (5.39)$$

the truncation would occur for $\phi \neq 0$ in a similar way to the models of §5.1 and §5.2 but with

$$k^* = \frac{1}{2} \left(\prod_{i=1}^{F_A} (2k_i + 1) - 1 \right). \quad (5.40)$$

Unfortunately the expressions for R -charges, σ e.t.c. quickly become very unwieldy. A simple generalisation of Eq.(5.37) and Eq.(5.38) does not work.

6 A note on symmetrics

With a little effort the entirety of §5 is immediately applicable to models with symmetric representations of the gauge group instead of antisymmetrics (as was the case for the models of Ref.[14]). The only differences will be as follows.

- The dual gauge group is $\text{SU}(n)$ with

$$n = (2k^* + 1) F_Q + 4k^* - N \quad (6.1)$$

and k^* defined as before.

- The mesons P_j and \tilde{P}_j are in symmetric representations of the appropriate quark flavour groups instead of antisymmetric representations.
- The electric baryons go like

$$B_r = (\phi^\alpha S)^r (Q^2)^{N-r} \quad (6.2)$$

and magnetic baryons go like

$$b_r = (\phi^\alpha S)^{2(F_Q+2)k^*-r} (q^2)^{F_Q-N+r}. \quad (6.3)$$

This accounts for the fact that symmetric tensors cannot be contracted with a single ϵ . Instead one needs to use two. The baryon map is still $B_r \leftrightarrow b_r$.

- The R -charges of the electric and magnetic quarks are

$$R_Q = 1 - \frac{N - 2k^*}{F_Q(1 + k^*)} \quad \text{and} \quad R_q = 1 - \frac{n - 2k^*}{F_Q(1 + k^*)} \quad (6.4)$$

respectively. The R -charges of the mesons and the R -symmetry anomalies are updated accordingly.

The gauge singlet sector is completely oblivious to whether we use symmetric or antisymmetric representations of the gauge group in the theory. In particular, the values of α , σ and the constrained ρ 's will remain unchanged. We have checked this explicitly for the two symmetric model and it should apply to the rest of §5.

7 Conclusions

We have investigated the role that singlets can play in establishing new electric/magnetic Kutasov type dualities (i.e. with superpotentials). Our models are based on $\mathcal{N} = 1$ SQCD with F_Q flavours of quark and antiquark with additional fields in the adjoint, antisymmetric or symmetric representations of $SU(N)$. The singlets generate R -symmetry violating couplings once they acquire VEVs, and these couplings in turn allow for a simple truncation of the chiral ring. Consequently a much wider class of dualities can be investigated. On the other hand 't Hooft anomaly matching can be performed on the full R -symmetric theory and provides the usual acid test of the duality.

The central observation required to make the 't Hooft anomaly matching work was how to identify the correct singlet content of the elementary mesons in the magnetic theory. Once this has been achieved, it is possible to find electric/magnetic duals with any number of adjoints or antisymmetric or symmetric.

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